Different Trade Models, Different Trade Elasticities?

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ABSTRACT

How has the development of new trade models changed our understanding of the welfare gains from trade? Answering this question depends solely on estimates of the trade elasticity obtained using techniques applicable across different models. In this paper we build on the methods of Simonovska and Waugh (2011) and we develop a common estimator for the trade elasticity that is applicable across different models that feature micro-level heterogeneity. The benefit of our approach is that, while the estimation uses the same moment conditions, it allows for different micro structures to matter. We apply the estimator to the models of Eaton and Kortum (2002), Bernard, Eaton, Jensen, and Kortum (2003), and a variant of the framework of Melitz (2003) and Chaney (2008). We find that the trade elasticity estimates differ considerably across models. The results suggest that the Bernard, Eaton, Jensen, and Kortum (2003) model yields the highest, while the Melitz (2003) model yields the lowest welfare gains from trade.

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1. Introduction

Quantitative results from structural gravity models of international trade depend critically on the elasticity of trade with respect to trade frictions.[1] In particular, Arkolakis, Costinot, and Rodriguez-Clare (2011) show that the trade elasticity is one of only two statistics needed to measure the welfare cost of autarky in a large and important class of trade models. Hence, the authors argue that, for a given elasticity parameter value, different trade models yield the same welfare gains.

Arkolakis, Costinot, and Rodriguez-Clare's (2011) theoretical result implies that an estimator for the trade elasticity that is common across models is needed in order to recover the key elasticity parameter. In this paper, we build on the methodology developed by Simonovska and Waugh (2011) and we develop a common estimator for the trade elasticity that is applicable across different models that feature micro-level heterogeneity. First, we use observed bilateral trade flows to recover all sufficient parameters to simulate each model and to obtain trade flows and prices as functions of the parameter of interest. Then, we apply a simulated method of moments estimator that minimizes the distance between moments obtained from real data and from artificial data generated by each model. Hence, our estimator applies a common set of rules to different data-generating processes.

Using disaggregate price and trade-flow data for the year 2004 from the thirty largest economies in the world, we apply our estimator to three canonical models of trade with heterogeneity: the perfectly-competitive Ricardian model of Eaton and Kortum (2002), the imperfectly-competitive extension by Bernard, Eaton, Jensen, and Kortum (2003), and the monopolistically-competitive framework of Melitz (2003) as articulated in Chaney (2008). The exercise results in different elasticity estimates across the three models. The Bernard, Eaton, Jensen, and Kortum (2003) model yields an estimate of 2.81, the Eaton and Kortum (2002) model generates a value of 4.21, and the Melitz (2003) model suggests a range for the elasticity between 3.93 and 4.86, depending on the price accounting methodology employed. Since welfare is inversely related to the trade elasticity, the results imply that the Bernard, Eaton, Jensen, and Kortum (2003) model generates the highest, while the Melitz (2003) model potentially yields the lowest gains from trade.

Given the stark welfare implications arising from the exercise, it is important to understand why we obtain estimates of the parameter that differ across models. To apply our methodology, we first compute moments in the data that are informative about the elasticity of trade. In particular, using disaggregate price data across countries, we compute the first and the second

[1] The class of models includes Krugman (1980), Anderson and van Wincoop (2003), Eaton and Kortum (2002), Bernard, Eaton, Jensen, and Kortum (2003), and Melitz (2003) as articulated in Chaney (2008), which all generate log-linear relationships between bilateral trade flows and trade frictions.

order statistic of logged price differences across goods for each country-pair. Price differences between countries are meaningful because they reveal information about bilateral trade frictions. These approximation of bilateral trade barriers together with aggregate price and trade flow-data yield informative moments about the trade elasticity from a gravity equation.

We compute the same moments from artificial data generated by each model. The procedure involves using observed bilateral trade flows to recover all sufficient parameters to simulate each model and to obtain trade flows and prices as functions of the parameter of interest. In order to recover the elasticity parameter for each model, we minimize the distance between the moments obtained from real and from artificial data. Although the three models demand identical rules in order to compute the moments of interest, the moments behave differently as functions of the elasticity parameter across models. The key difference lies in the trade-barrier approximations generated by the three models.

Eaton and Kortum's (2002) perfectly-competitive model implies that goods' relative prices across countries equal relative marginal costs of production and delivery of the lowest-cost suppliers to each market. No-arbitrage arguments bound the objects above by trade barriers.

Relative prices and costs are bounded above by trade barriers in the model of Bernard, Eaton, Jensen, and Kortum (2003) as well. For a given good, however, the relative price may lie above or below the relative cost because mark-ups are non-constant in this imperfectly-competitive environment. The crucial observation is that mark-ups are negatively related to costs. As a result, logged cost differences stochastically dominate logged price differences. Consequently, the expectations of first and second order statistics of logged cost differences are higher than their counterparts obtained from logged price differences. Trade-barrier approximations are therefore lower in Bernard, Eaton, Jensen, and Kortum's (2003) relative to Eaton and Kortum's (2002) model. The larger bias in trade-barrier estimates obtained from Bernard, Eaton, Jensen, and Kortum's (2003) model means that the model calls for a lower trade elasticity in order to match the moments in the data.

In the Ricardian models above, the maximum operator over a finite sample of prices underestimates the trade cost with positive probability and overestimates the trade cost with zero probability. Consequently, the maximum price ratio lies strictly below the true trade cost, in expectation. The argument does not apply to the monopolistically-competitive framework of Melitz (2003), where different countries supply different varieties of a good to different destinations. The relative price of a good across countries may over- or under-estimate the bilateral trade barrier, depending on the particular variety (or set thereof) used in order to compute the price ratio. Hence, in expectation, the trade-barrier approximations are higher in the Melitz (2003) model than in the Ricardian frameworks. The smaller bias in trade-barrier estimates requires a higher trade elasticity to match the moment in the data.

In sum, although the models share identical estimating equations for the elasticity parameter, the underlying market structure that differs across the models results in different trade elasticity estimates. Therefore, the three models yield different welfare gains from trade.

The models share three properties that enable us to estimate the trade elasticity and quantify the welfare gains from trade. First, consumer preferences over varieties are of the constant elasticity of substitution (CES) kind. Second, the production technologies feature heterogeneity among the suppliers of goods. Third, the particular parameterization of the productivity distribution in each framework allows it to generate a gravity equation of trade that is log-linear in bilateral trade barriers. These assumptions ensure that the models generate identical formulas for the welfare gains from trade as well as identical estimating equations for the key elasticity parameter.

Two alternative common estimators may be used in order to estimate the elasticity of trade in these models. The first is the approach proposed by Eaton and Kortum (2002). Simonovska and Waugh (2011) demonstrate that this methodology yields biased estimates of the trade elasticity in finite samples of price data and that quantitatively, the bias is severe.

The second approach uses data on changes in bilateral trade flows and tariffs during trade liberalization episodes, as in Head and Ries (2001), Romalis (2007), and Caliendo and Parro (2011). The approach typically associates the entire change in trade flows during a trade liberalization with changes in tariffs. This results in high estimates of the trade elasticity, since changes in non-tariff barriers that occur during trade liberalizations are not accounted for in the estimation. Moreover, instrumental-variable methods aimed to alleviate the omitted-variables bias in the estimation yield a wide range of estimates for the trade elasticity. Yet, a clear guide toward choosing the best control variables is not available because the estimation does not tailor to a particular structural model.

We view this approach as complementary to ours. One merit of our method relative to the alternative is that our approach accounts for the micro-level heterogeneity of each model in a transparent and structural way. This is important in light of our results which suggest that differences in the micro-structures of trade models are responsible for different elasticity estimates. Since the elasticity of trade governs welfare in each model, understanding how the micro-structure of new models of heterogeneity relates to the trade elasticity is key in order to understand the sources of the welfare gains from trade.

2. Three Canonical Models of Trade

2.1. Eaton and Kortum (2002)

We outline the environment of the multi-country Ricardian model of trade introduced by Eaton and Kortum (2002). We consider a world with N countries, where each country has a tradable final-goods sector. There is a continuum of tradable goods indexed by $j \in [0, 1]$.

Within each country i, there is a measure of consumers L_i. Each consumer has one unit of time supplied inelastically in the domestic labor market and enjoys the consumption of a CES bundle of final tradable goods with elasticity of substitution $\rho > 1$

$$U_i = \left[\int_0^1 x_i(j)^{\frac{\rho-1}{\rho}} dj \right]^{\frac{\rho}{\rho-1}}.$$

To produce quantity $x_i(j)$ in country i, a firm employs labor using a linear production function with productivity $z_i(j)$. Country i's productivity is, in turn, the realization of a random variable (drawn independently for each j) from its country-specific Fréchet probability distribution

$$F_i(z_i) = \exp(-T_i z_i^{-\theta}).$$

The country-specific parameter $T_i > 0$ governs the location of the distribution; higher values of it imply that a high productivity draw for any good j is more likely. The parameter $\theta > 1$ is common across countries and, if higher, it generates less variability in productivity across goods.

Having drawn a particular productivity level, a perfectly competitive firm from country i incurs a marginal cost to produce good j of $w_i/z_i(j)$, where w_i is the wage rate in the economy. Shipping the good to a destination n further requires a per-unit iceberg trade cost of $\tau_{ni} > 1$ for $n \neq i$, with $\tau_{ii} = 1$. We assume that cross-border arbitrage forces effective geographic barriers to obey the triangle inequality: For any three countries i, k, n, $\tau_{ni} \leq \tau_{nk}\tau_{ki}$.

Perfect competition forces the price of good j from country i to destination n to be equal to the marginal cost of production and delivery

$$p_{ni}(j) = \frac{\tau_{ni} w_i}{z_i(j)}.$$

So, consumers in destination n would pay $p_{ni}(j)$, should they decide to buy good j from i.

Consumers purchase good j from the low-cost supplier; thus, the actual price consumers in n pay for good j is the minimum price across all sources k

$$p_n(j) = \min_{k=1,\dots,N} \left\{ p_{nk}(j) \right\}.$$

2.2. Bernard, Eaton, Jensen, and Kortum (2003)

Bernard, Eaton, Jensen, and Kortum (2003) introduce Bertrand competition into Eaton and Kortum's (2002) model. The most important implication from this extension is that individual good prices differ from the Eaton and Kortum (2002) model.

Let $c_{kni}(j) \equiv \tau_{ni} w_i / z_{ki}(j)$ be the cost that the k-th most efficient producer of good j in country i faces in order to deliver a unit of the good to destination n. With Bertrand competition, as with perfect competition, the low-cost supplier of each good serves the market. For good j in market n, this supplier has the following cost $c_{1n}(j) = \min_i \{c_{1ni}(j)\}$. This supplier is constrained not to charge more than the second-lowest cost of supplying the market, which is $c_{2n} = \min \{c_{2ni^*}(j), \min_{i \neq i^*} \{c_{1ni}(j)\}\}$, where i^* satisfies $c_{1ni^*}(j) = c_{1n}(j)$. Hence, the price of good j in market n is

$$p_n(j) = \min \{c_{2n}(j), \bar{m} c_{1n}(j)\},$$

where $\bar{m} = \rho/(\rho - 1)$.

Finally, for each country i, productivity, $z_{ki}(j)$ for $k = 1, 2$ is drawn from

$$F_i(z_1, z_2) = \left[1 + T_i(z_2^{-\theta} - z_1^{-\theta})\right] \exp\left(-T_i z_2^{-\theta}\right).$$

2.3. Melitz (2003)

In this section, we outline a variant of the Melitz (2003) model parameterized as in Chaney (2008). In the exposition, we follow closely Eaton, Kortum, and Kramarz (2011).

Consider a world of N countries engaged in trade of final goods, where N is finite. Let i represent an exporter and n an importer, that is, i is the source country, while n is the destination country.

Let there be a fixed measure of firms with efficiency of at least z in i:

$$\mu_i(z) = T_i z^{-\theta}, \qquad z > 0$$

Let the marginal cost of firm with efficiency z of producing a good in i and delivering it to n be

$$c_{ni}(z) = \frac{w_i \tau_{ni}}{z}$$

Then

$$\mu_{ni}(c) = \Phi_{ni} c^{\theta}, \qquad \Phi_{ni} = T_i (w_i \tau_{ni})^{-\theta}$$

Similarly we can define the pdf of firms,

$$d\mu_{ni}(c) = \theta\Phi_{ni}c^{\theta-1}$$

and the measure of firms from i that successfully operate in n,

$$\mu_{ni}(\bar{c}_{ni}) = \Phi_{ni}\bar{c}_{ni}^{\theta}, \tag{1}$$

where $\bar{c}_{ni}$ is the maximum cost a firm can have in order to sell to n.

Since there is no entry and exit, firms in country i earn positive profits, Π_i, that are distributed to consumers in i, whose measure is L_i. Per-consumer profit is $\pi_i = \Pi_i/L_i$.

Consumers supply their unit labor endowment to the market and earn a wage of w_i. Consumer income is $y_i = w_i + \pi_i$. Consumer preferences are CES over varieties of goods produced in all source countries. $\rho > 1$ is the elasticity of substitution.

Firms solve a per-market profit maximization problem. In order to serve market n firms incur a fixed cost f_n, expensed in destination wages, $w_n f_n$.

Let CES demand for variety of good produced by firm from i with cost c_{ni} in market n be

$$q_{ni}(c_{ni}) = y_n L_n \frac{(p_{ni}(c_{ni}))^{-\rho}}{\bar{P}_n^{1-\rho}},$$

where

$$\bar{P}_n^{1-\rho} = \sum_{k=1}^{N} \int_0^{\bar{c}_{nk}} (p_{nk}(c_{nk}))^{1-\rho} d\mu_{nk}(c_{nk}) \tag{2}$$

The price that the firm from i charges in n is

$$p_{ni}(c_{ni}) = \bar{m}c_{ni}$$

Firm sales and profits in market n are

$$r_{ni}(c_{ni}) = y_n L_n \left(\frac{\rho}{\rho-1}\right)^{1-\rho} \left(\frac{c_{ni}}{\bar{P}_n}\right)^{1-\rho}$$
$$\pi_{ni}(c_{ni}) = y_n L_n \left(\frac{\rho}{\rho-1}\right)^{-\rho} \frac{1}{\rho-1} \left(\frac{c_{ni}}{\bar{P}_n}\right)^{1-\rho} - w_n f_n$$

Cost cutoff for firms from i to sell to n, $\bar{c}_{ni}$ satisfies $\pi(\bar{c}_{ni}) = 0$ and is given by

$$\bar{c}_{ni} = \left[\frac{w_n f_n}{y_n L_n} \frac{\bar{P}_n^{1-\rho}}{\left(\frac{\rho}{\rho-1}\right)^{-\rho} \frac{1}{\rho-1}} \right]^{\frac{1}{1-\rho}}$$

Since $\bar{c}_{ni} = \bar{c}_{nn} \; \forall i$, define cutoffs as $\bar{c}_n$.

2.4. Trade Flows, Aggregate Prices, and Welfare

In this section, we demonstrate under what assumptions trade flows, price indices (up to a constant scalar), and welfare are identical in the three models.

The pricing rule and the productivity distribution in the Eaton and Kortum (2002) model allow us to obtain the following CES exact price index for each destination n

$$P_n^{ek} = \gamma_1 \Phi_n^{-\frac{1}{\theta}}, \quad \text{where} \quad \Phi_n = \left[\sum_{k=1}^{N} T_k (\tau_{nk} w_k)^{-\theta} \right]. \tag{3}$$

In the above equation, $\gamma_1 = \left[\Gamma\left(\frac{\theta+1-\rho}{\theta}\right)\right]^{\frac{1}{1-\rho}}$ is the Gamma function, and parameters are restricted such that $\theta > \rho - 1$.

To calculate trade flows between countries, let X_n be country n's expenditure on final goods, of which X_{ni} is spent on goods from country i. Since there is a continuum of goods, computing the fraction of income spent on imports from i, X_{ni}/X_n, can be shown to be equivalent to finding the probability that country i is the low-cost supplier to country n given the joint distribution of efficiency levels, prices, and trade costs for any good j. The expression for the share of expenditures that n spends on goods from i or, as we will call it, the trade share is

$$\frac{X_{ni}}{X_n} = \frac{T_i (\tau_{ni} w_i)^{-\theta}}{\sum_{k=1}^{N} T_k (\tau_{nk} w_k)^{-\theta}}. \tag{4}$$

The derivation is similar for the Bernard, Eaton, Jensen, and Kortum (2003) model. The pricing rule and the productivity distribution in the Bernard, Eaton, Jensen, and Kortum (2003) model allow us to obtain the following CES exact price index for each destination n

$$P_n^{bejk} = \gamma_2 \Phi_n^{-\frac{1}{\theta}}. \tag{5}$$

In the above equation, $\gamma_2 = \left[\frac{1+\theta-\rho+(\rho-1)\bar{m}^{-\theta}}{1+\theta-\rho} \Gamma\left(\frac{2\theta+1-\rho}{\theta}\right)\right]^{\frac{1}{1-\rho}}$.

Finally, to derive trade flows from i to n in the Melitz model, integrate over all firms from i that serve the particular destination,

$$X_{ni} = \int_0^{\bar{c}_n} r_{ni}(c_{ni}) d\mu_{ni}(c_{ni})$$
$$= \theta \Phi_{ni} y_n L_n \left(\frac{\rho}{\rho - 1}\right)^{1-\rho} \frac{1}{1 - \rho + \theta} \frac{\bar{c}_n^{1-\rho+\theta}}{\bar{P}_n^{1-\rho}}.$$

Trade share of i in n is

$$X_{ni}/X_n = \frac{\Phi_{ni}}{\sum_k \Phi_{nk}},$$

which is identical to the previous two models given the definition of Φ_{ni}.

Using equilibrium prices and measures of firms in (2) yields

$$\bar{P}_n = \left\{ \frac{\theta}{1 - \rho + \theta} \left[\frac{\rho}{\rho - 1}\right]^{1-\rho} \frac{1}{\left[\left(\frac{\rho}{\rho-1}\right)^{-\rho} \frac{1}{\rho-1}\right]^{\frac{1-\rho+\theta}{1-\rho}}} \left[\frac{w_n f_n}{y_n L_n}\right]^{\frac{1-\rho+\theta}{1-\rho}} \sum_k \Phi_{nk} \right\}^{-\frac{1}{\theta}}.$$

Thus, in order for the Melitz model to generate a price index that is a constant multiple of the Eaton and Kortum (2002) (or the Bernard, Eaton, Jensen, and Kortum (2003)) price index, it is sufficient that $w_n f_n \propto y_n L_n$. First, we demonstrate that $w_n \propto y_n$.

Use the price index in the cutoff to obtain

$$\bar{c}_n = \left[\frac{w_n f_n}{y_n L_n} \left(\frac{\theta \rho}{1 - \rho + \theta}\right) \sum_k \Phi_{nk}\right]^{-\frac{1}{\theta}}. \tag{6}$$

To see that $w_n \propto y_n$, first notice that the expenditure on fixed costs to reach n by all firms from i that sell to n is a constant share of these firms' sales

$$\frac{w_n f_n \Phi_{ni} (\bar{c}_{ni})^\theta}{X_{ni}} = \frac{1 - \rho + \theta}{\theta \rho}.$$

Moreover, the share of variable profits of firms from i who sell to n out of the sales of these firms in n is also constant and equal to $1/\rho$. Since profits of firms from i generated in market n are the difference between the firms' variable profits generated in that market and the fixed costs incurred to access it, it must be that profits are also a constant share out of sales. The share is given by $1/\rho - (1 - \rho + \theta)/(\theta\rho) = (\rho - 1)/(\theta\rho)$. Then, $\sum_i \pi_{in} = (\rho - 1)/(\theta\rho) \sum_i X_{in}$. By trade balance $\sum_i X_{in} = \sum_i X_{ni}$. Then, income-spending equality ensures that $y_n L_n = \sum_i X_{ni}$. Hence the share of total profits of firms from n out of total income in n is $(\rho - 1)/(\theta\rho)$. The share of labor income out of total income is then $1-(\rho-1)/(\theta\rho)=(\theta\rho-\rho+1)/(\theta\rho)$. Similarly, in per capita terms, $w_n/y_n=(\theta\rho-\rho+1)/(\theta\rho)$.

Thus, if we assume that $f_n \propto L_n$, with proportionality constant $A>0$, then $\bar{c}_n$ and $\bar{P}_n$ will be proportional to $\Phi_n^{-\frac{1}{\theta}}$. Hence, the Melitz model would yield,

$$P_n^{mel}=\gamma_3\Phi_n^{-\frac{1}{\theta}}, \tag{7}$$

where $\gamma_3=\left\{\frac{\theta}{1-\rho+\theta}\left[\frac{\rho}{\rho-1}\right]^{1-\rho}\frac{1}{\left[\left(\frac{\rho}{\rho-1}\right)^{-\rho}\frac{1}{\rho-1}\right]^{\frac{1-\rho+\theta}{1-\rho}}}\left[\frac{(\theta\rho-\rho+1)A}{\theta\rho}\right]^{\frac{1-\rho+\theta}{1-\rho}}\right\}^{-\frac{1}{\theta}}$.

Expressions (4) and (3) (alternatively (5) or (7)) allow us to relate trade shares to trade costs and the price indices of each trading partner via the following equation

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} \;=\; \frac{\Phi_i}{\Phi_n}\tau_{ni}^{-\theta}=\left(\frac{P_i\tau_{ni}}{P_n}\right)^{-\theta}, \tag{8}$$

where $\frac{X_{ii}}{X_i}$ is country i's expenditure share on goods from country i, or its home trade share, and P_n is country n's price index, descaled by either constant $\gamma_1,\gamma_2,\gamma_3$.

Finally, it is easy to show that the welfare gains from trade are essentially captured by changes in the CES price index that a representative consumer faces. Because of the tight link between prices and trade shares, the models generate the following relationship between changes in price indices and changes in home trade shares, as well as, the elasticity parameter:

$$\frac{P_n'}{P_n}-1=1-\left(\frac{X_{nn}'/X_n'}{X_{nn}/X_n}\right)^{\frac{1}{\theta}},$$

where the left-hand side can be interpreted as the percentage compensation a representative consumer in country n requires to move between two trading equilibria.

2.5. The Elasticity of Trade

The key parameter determining trade flows (equation (8)) and welfare (equation (9)) is θ. To see the parameter's importance for trade flows, take logs of equation (8) yielding

$$\log\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right) \;=\; -\theta\left[\log\left(\tau_{ni}\right)-\log(P_i)+\log(P_n)\right]. \tag{9}$$

As this expression makes clear, θ controls how a change in the bilateral trade costs, τ_{ni}, will change bilateral trade between two countries. This elasticity is important because if one wants to understand how a bilateral trade agreement will impact aggregate trade or to simply understand the magnitude of the trade friction between two countries, then a stand on this elasticity is necessary. This is what we mean by the elasticity of trade.

To see the parameter's importance for welfare, it is easy to demonstrate that (9) implies that θ represents the inverse of the elasticity of welfare with respect to domestic expenditure shares

$$\log(P_n) = -\frac{1}{\theta}\log\left(\frac{X_{nn}}{X_n}\right).$$

Hence, decreasing the domestic expenditure share by one percent generates a $(1/\theta)/100$-percent increase in consumer welfare. Thus, in order to measure the impact of trade policy on welfare, it is sufficient to obtain data on realized domestic expenditures and an estimate of the elasticity of trade.

Given θ's impact on trade flows and welfare, this elasticity is absolutely critical in any quantitative study of international trade.

3. Estimating θ

There are three ways to estimate the parameter θ in (8). One approach involves using changes in trade flows and tariffs. The merit of our methodology relative to this alternative approach were discussed in the introduction.

Equation (8) suggests a second approach to estimate θ, if one had data on trade shares, aggregate prices, and trade costs. The key issue is that trade costs are not observed. In what follows, we explain how Eaton and Kortum (2002) approximate trade costs and aggregate prices in order to estimate θ. Then, we outline our simulated-method of moments estimator, which builds on Simonovska and Waugh (2011), and uses moments from Eaton and Kortum's (2002) methodology in order to back out θ in each model.

3.1. Approximating Trade Costs

The main problem with estimating θ is that one must disentangle θ from trade costs, which are not observed. Eaton and Kortum (2002) propose approximating trade costs using *disaggregate* price information across countries. In particular, in the Eaton and Kortum (2002) model, the maximum price difference across goods between two countries bounds the bilateral trade cost, which solves the indeterminacy issue.

To illustrate this argument, suppose that we observe the price of good ℓ across locations, but we do not know its country of origin.[2] We know that the price of good ℓ in country n relative to

[2] This is the most common case, though Donaldson (2009) exploits a case where he knows the place of origin for one particular good, salt. He argues convincingly that in India, salt was produced in only a few locations and exported everywhere; thus, the relative price of salt across locations identifies the trade friction.

country i must satisfy the following inequality

$$\frac{p_n(\ell)}{p_i(\ell)} \leq \tau_{ni}. \tag{10}$$

That is, the relative price of good ℓ must be less than or equal to the trade friction. This inequality must hold because if it does not, then $p_n(\ell) > \tau_{ni} p_i(\ell)$ and an agent could import ℓ at a lower price. Thus, the inequality in (10) places a lower bound on the trade friction.

Improvements on this bound are possible if we observe a sample of L goods across locations. This follows by noting that the *maximum* relative price must satisfy the same inequality

$$\max_{\ell \in L} \left\{ \frac{p_n(\ell)}{p_i(\ell)} \right\} \leq \tau_{ni}. \tag{11}$$

This suggests a way to exploit *disaggregate* price information across countries and to arrive at an estimate of τ_{ni} by taking the maximum of relative prices over goods. Thus, Eaton and Kortum (2002) approximate τ_{ni}, in logs, by

$$\log \hat{\tau}_{ni}(L) = \max_{\ell \in L} \left\{ \log\left(p_n(\ell)\right) - \log\left(p_i(\ell)\right) \right\}, \tag{12}$$

where the "hat" denotes the approximated value of τ_{ni} and (L) indexes its dependence on the sample size of prices.

3.2. Estimating the Elasticity

Given the approximation of trade costs, Eaton and Kortum (2002) derive an econometric model that corresponds to (9). For a sample of L goods, they estimate a parameter, β, using a method of moments estimator, which takes the ratio of the average of the left-hand side of (9) to the average of the term in the square bracket of the right-hand side of (9), where the averages are computed across all country pairs. Mathematically, their estimator is

$$\hat{\beta} = -\frac{\sum_n \sum_i \log\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right)}{\sum_n \sum_i \left(\log \hat{\tau}_{ni}(L) + \log \hat{P}_i - \log \hat{P}_n\right)}, \tag{13}$$

$$\text{where} \quad \log \hat{\tau}_{ni}(L) = \max_{\ell \in L} \left\{ \log p_n(\ell) - \log p_i(\ell) \right\},$$

$$\text{and} \quad \log \hat{P}_i = \frac{1}{L} \sum_{\ell=1}^{L} \log(p_i(\ell)).$$

The value of β is EK's preferred estimate of the elasticity θ. Throughout, we will denote by $\hat{\beta}$ the estimator defined in equation (13) to distinguish it from the value θ. As discussed, the second line of expression (13) approximates the trade cost. The third line approximates the aggregate price indices. The top line represents a rule that combines these statistics, together with observed trade flows, in an attempt to estimate the elasticity of trade.

3.3. Simonovska and Waugh (2011) Estimator

Simonovska and Waugh (2011) show that EK's estimator of the trade elasticity is biased in any finite sample of goods' prices. In particular, assuming that price and trade data are generated from the Eaton and Kortum (2002) model, the expected maximum log price difference is biased below the true log trade cost

$$\mathbb{E}\left(\max_{\ell \in L}\{\log p_n(\ell) - \log p_i(\ell)\}\right) < \tau_{ni}$$

The intuition for the result is as follows. Relative prices of goods are bounded above by trade barriers, so the maximum operator over a finite sample of prices underestimates the trade cost with positive probability and overestimates the trade cost with zero probability. Consequently, the maximum price difference lies strictly below the true trade cost, in expectation. This necessarily implies that any estimate β obtained from the rule $\hat{\beta}$ lies strictly above the elasticity parameter θ, in expectation,

$$\mathbb{E}(\hat{\beta}) > \theta.$$

The authors then propose to use the moment β from Eaton and Kortum's (2002) approach and to recover the parameter value for θ via a simulation of the Eaton and Kortum (2002) model. The procedure successfully recovers the parameter value for θ because Eaton and Kortum's (2002) estimator is biased but monotone, and nearly proportional to θ. Thus, using this informative moment as a basis for estimation, the procedure involves the following steps:

A. Recover necessary parameters (except θ) from trade data.

B. Estimate $\beta(\theta)$ using artificial data and compare to β from real data.

C. Update θ until $\beta(\theta)$ is "close" to β.

This procedure naturally extends to the models of Bernard, Eaton, Jensen, and Kortum (2003) and Melitz (2003) outlined earlier. In particular, assuming that the price and trade flow data were generated from either of these two models, one can repeat the three steps above and obtain estimates of the trade elasticity that are consistent with these two models. In the sections that follow, we outline the simulation procedure as it applies to each model and we report estimates of the trade elasticity.

3.4. Simulation Approach

In this subsection, we follow the exposition in Simonovska and Waugh (2011), which applied to the Eaton and Kortum (2002) model, and we demonstrate how to recover all parameters of interest in the Bernard, Eaton, Jensen, and Kortum (2003) and Melitz (2003) models up to the unknown scalar θ from trade data only. Then we describe our simulation approach. This provides the foundation for the simulated method of moments estimator that we propose.

Step 1.—We estimate the parameters for the country-specific productivity distributions and trade costs from bilateral trade-flow data. We follow closely the methodologies proposed by Eaton and Kortum (2002) and Waugh (2010b). First, we derive the gravity equation from expression (4) by dividing the bilateral trade share by the importing country's home trade share,

$$\log\left(\frac{X_{ni}/X_n}{X_{nn}/X_n}\right) = S_i - S_n - \theta \log \tau_{ni}, \tag{14}$$

where S_i is defined as $\log\left[T_i w_i^{-\theta}\right]$. Note that (14) is a different equation than expression (8), which is derived by dividing the bilateral trade share by the exporting country's home trade share, and is used to estimate θ. S_i's are recovered as the coefficients on country-specific dummy variables given the restrictions on how trade costs can covary across countries. Following the arguments of Waugh (2010b), trade costs take the following functional form

$$\log(\tau_{ni}) = d_k + b_{ni} + ex_i + \nu_{ni}. \tag{15}$$

Here, trade costs are a logarithmic function of distance, where d_k with $k = 1, 2, ..., 6$ is the effect of distance between country i and n lying in the k-th distance interval.[3] b_{ni} is the effect of a shared border in which $b_{ni} = 1$ if country i and n share a border and zero otherwise. The term ex_i is an exporter fixed effect and allows for the trade-cost level to vary depending upon the exporter. We assume that ν_{ni} reflects other factors and is orthogonal to the regressors and normally distributed with mean zero and standard deviation σ_ν. We use least squares to estimate equations (14) and (15).

Step 2.—The parameter estimates obtained from the first-stage gravity regression are sufficient to simulate trade flows and micro-level prices in each model up to a constant, θ.

The relationship is obvious in the estimation of trade barriers since $\log(\tau_{ni})$ is scaled by θ in (14).

[3] Intervals are in miles: $[0, 375)$; $[375, 750)$; $[750, 1500)$; $[1500, 3000)$; $[3000, 6000)$; and $[6000, \text{maximum}]$. An alternative to specifying a trade-cost function is to recover scaled trade costs as a residual using equation (8), trade data, and measures of aggregate prices as in Waugh (2010a).

Step 2a.— To simulate micro-level prices in the Eaton and Kortum (2002), see Simonovska and Waugh (2011).

Step 2b.— To see that we can simulate micro-level prices in the Bernard, Eaton, Jensen, and Kortum (2003) model as a function of θ only, we draw on an argument in Bernard, Eaton, Jensen, and Kortum (2003). To simulate their model, Bernard, Eaton, Jensen, and Kortum (2003) reformulate the model in terms of efficiency. In particular, given two productivity draws for good j in country i, $z_{1i}(j)$ and $z_{2i}(j)$, they define the following objects

$$u_{1i}(j) = T_i z_{1i}(j)^{-\theta}$$
$$u_{2i}(j) = T_i z_{2i}(j)^{-\theta}$$

Bernard, Eaton, Jensen, and Kortum (2003) demonstrate that these objects are distributed according to

$$Pr[u_{1i} \leq u_1] = 1 - \exp(-u_1)$$
$$Pr[u_{2i} \leq u_2 | u_{1i} = u_1] = 1 - \exp(-u_2 + u_1)$$

To simulate trade flows and prices from the model, we define the following variables

$$v_{1i}(j) = \left(\frac{u_{1i}(j)}{T_i w_i^{-\theta}}\right)$$
$$v_{2i}(j) = \left(\frac{u_{2i}(j)}{T_i w_i^{-\theta}}\right)$$

Let $\tilde{S}_i = \exp\{S_i\}$, with $S_i = \log(T_i w_i^{-\theta})$ coming from gravity. Applying the pdf transformation rule, it is easy to demonstrate that $v_{1i}(j)$ is distributed according to

$$Pr[v_{1i} \leq v_1] = 1 - \exp(-\tilde{S}_i v_1) \tag{16}$$

Similarly,

$$Pr[v_{2i} \leq v_2 | v_{1i} = v_1] = 1 - \exp(-\tilde{S}_i v_2 + \tilde{S}_i v_1) \tag{17}$$

Thus, to simulate the model we draw minimum unit costs from (16), and conditional on these draws, we draw the second lowest unit costs from (17). Hence, having obtained the coefficients S_i from the first-stage gravity regression, we can simulate the inverse of marginal costs and prices.

To simulate the model, we assume that there are a large number (100,000) of potentially tradable goods. For each country, the inverse marginal costs are drawn from the country-specific distributions above and assigned to each good. Then, for each importing country and each good, the two lowest-cost suppliers across countries are found, realized prices are recorded, and aggregate bilateral trade shares are computed. From the realized prices, a subset of goods common to all countries is defined and the subsample of prices is recorded – i.e., we are acting as if we were collecting prices for the international organization that collects the data.

Step 2c.— The simulation of the Melitz (2003) model is more intricate. In the Eaton and Kortum (2002) and Bernard, Eaton, Jensen, and Kortum (2003) models, a good is indexed by $j \in [0, 1]$ or an integer when discretized on the computer. In the Melitz (2003) model, different countries are consuming and producing different goods and varieties of these goods. We define a good as an integer on the computer and a variety of the good is defined by the country of origin. There could be up to N suppliers of a variety of that good. Once again, we can simulate the prices of these varieties easily.

After we discretize the continuum and we define a set of goods, we determine the subset of goods produced domestically for each country. Then, we simulate inverse marginal costs for each good and variety thereof in each country. Inverse marginal costs are determined following Eaton, Kortum, and Kramarz (2011) who show how normalized inverse marginal costs can be sampled from a parameter free uniform distribution. After the appropriate re-scaling, the measure of goods producers with efficiency of at least z is given by (1).

Then, we determine the set of exported varieties for all country-pairs and we compute their prices. To derive relative prices of goods, we use two different rules. First, we let the price of a good in a country be the geometric average of prices across all varieties of that good sold in the country. Second, we randomly select a variety of a good sold in a country and we assign its price to the price of the good. Finally, we compute price indices and trade shares.

Step 3.—We added disturbances to the predicted trade shares with the disturbances drawn from a mean zero normal distribution with the standard deviation set equal to the standard deviation of the residuals from Step 1.

These steps then provide us with an artificial data set of micro-level prices and trade shares that mimic their analogs in the data. Given this artificial data set, we can then compute moments—as functions of θ—and compare them to the moments in the data.

3.5. Estimation

We perform an overidentified estimation with two moments. Below, we describe the moments we try to match and the details of our estimation procedure.

Moments. Define $\hat{\beta}_k$ as EK's method of moment estimator defined in (13) using the kth-order statistic over micro-level price differences. Then, the moments that we are interested in are

$$\hat{\beta}_k = -\frac{\sum_n \sum_i \log\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right)}{\sum_n \sum_i \left(\log \hat{\tau}^k_{ni}(L) + \log \hat{P}_i - \log \hat{P}_n\right)}, \quad k = 1, 2 \tag{18}$$

where $\hat{\tau}^k_{ni}(L)$ is computed as the kth-order statistic over L micro-level price differences between countries n and i.

We denote the simulated moments by $\beta_1(\theta_m, u_s|m)$ and $\beta_2(\theta_m, u_s|m)$, which come from the analogous formula as in (18) and are estimated from artificial data generated from each model by following **Steps 1-3** above. Note that these moments are a function of θ_m, where m denotes the possibility of a model-specific value, and depend upon a vector of random variables u_s associated with a particular simulation s. There are three components to this vector. First, there are the random productivity draws for production technologies for each good and each country. The second component is the set of goods sampled from all countries. The third component mimics the residuals ν_{ni} from equation (14), which are described in Section 3.4.

Stacking our data moments and averaged simulation moments gives us the following zero function

$$y(\theta_m) = \begin{bmatrix} \beta_1 - \frac{1}{S}\sum_{s=1}^{S} \beta_1(\theta_m, u_s|m) \\ \\ \beta_2 - \frac{1}{S}\sum_{s=1}^{S} \beta_2(\theta_m, u_s|m) \end{bmatrix}. \tag{19}$$

Estimation Procedure. We base our estimation procedure on the moment condition

$$E\left[y(\theta_o)\right] = 0,$$

where θ_{om} is the true value of θ_m. Thus, our simulated method of moments estimator is

$$\hat{\theta}_m = \arg\min_{\theta_m} \left[y(\theta_m)' \, \mathbf{W} \, y(\theta_m)\right],$$

where **W** is a 2×2 weighting matrix that we discuss below.

The idea behind this moment condition is that, though $\hat{\beta}_1$ and $\hat{\beta}_2$ will be biased away from θ, the moments $\beta_1(\theta_m, u_s|m)$ and $\beta_2(\theta_m, u_s|m)$ will be biased by the same amount when evaluated at θ_{om}, in expectation. Viewed in this language, our moment condition is closely related to the estimation of bias functions discussed in MacKinnon and Smith (1998) and to indirect inference, as discussed in Smith (2008). The key issue in MacKinnon and Smith (1998) is how the bias function behaves. As we argued earlier the bias is monotonic in the parameter of interest. Furthermore, Figure 1 below shows that the bias is basically linear, so it is well behaved.

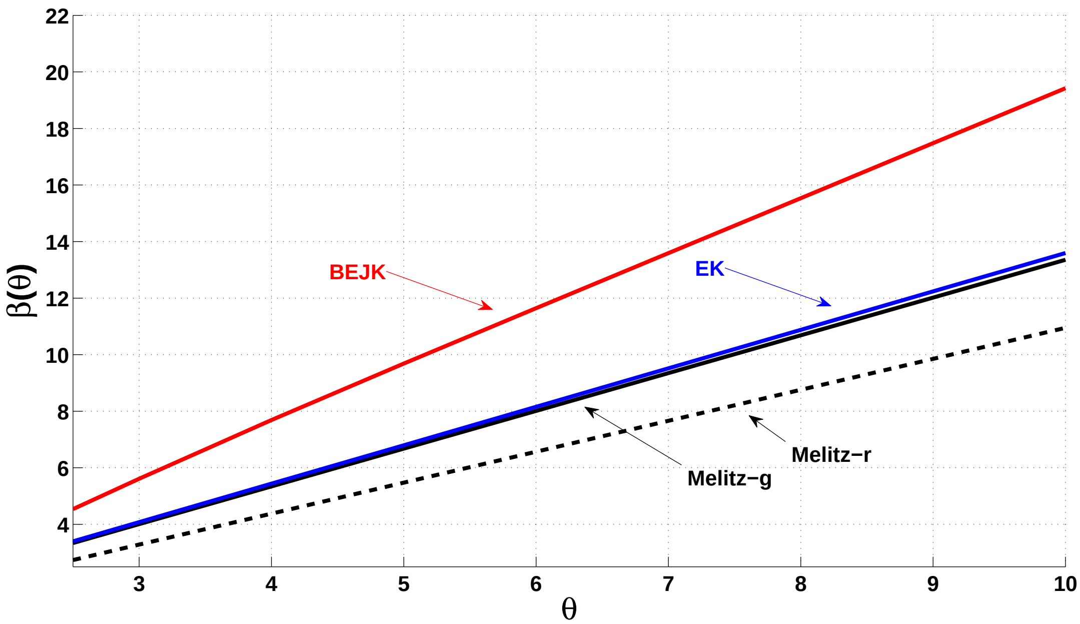

Figure 1: First Moment and Parameter of Interest

For the weighting matrix, we use the optimal weighting matrix suggested by Gouriéroux and Monfort (1996) for simulated method of moments estimators. Because the weighting matrix depends on our estimate of θ_m, we use a standard iterative procedure outlined in the next steps.

Step 4.—We make an initial guess of the weighting matrix $\mathbf{W}^0$ and solve for $\hat{\theta}_m^0$. Then, given this value we simulate the model to generate a new estimate of the weighting matrix.[4] With the new estimate of the weighting matrix we solve for a new $\hat{\theta}_m^1$. We perform this iterative procedure until our estimates of the weighting matrix and $\hat{\theta}_m$ converge. We explicitly consider simulation error because we utilize the weighting matrix suggested by Gouriéroux and Monfort (1996).

Step 5.—We compute standard errors using a bootstrap technique. We compute residuals from the data and the fitted values obtained using the estimates in (18), we resample the residuals with replacement, and we generate a new set of data using the fitted values. Using the data constructed from each resampling b, we computed new estimates β_1^b and β_2^b.

For each bootstrap b, we replace the moments β_1 and β_2 with bootstrap-generated moments β_1^b and β_2^b. To account for simulation error, a new seed is set to generate a new set of model-generated moments. Defining $y^b(\theta_m)$ as the difference in moments for each b, as in (19), we

[4]The computation of this matrix is described in Gouriéroux and Monfort (1996).

solve for

$$\hat{\theta}_m^b = \arg\min_{\theta_m} \left[y^b(\theta_m)' \, \mathbf{W} \, y^b(\theta_m) \right].$$

We repeat this exercise 100 times and we compute the standard error of our estimate of $\hat{\theta}$ as

$$\text{S.E.}(\hat{\theta}_m) = \left[\frac{1}{100} \sum_{b=1}^{100} (\hat{\theta}_m^b - \hat{\theta}_m)(\hat{\theta}_m^b - \hat{\theta}_m)' \right]^{\frac{1}{2}}.$$

This procedure for constructing standard errors is similar in spirit to the approach of Eaton, Kortum, and Kramarz (2011), who use a simulated method of moments estimator to estimate the parameters of a trade model featuring micro-level heterogeneity from the performance of French exporters.

3.6. Performance on Simulated Data

Figure 1 in the previous section plots values for the moment $\beta_1(\theta_m)$ obtained from simulations of each model as we varied θ_m. It is clear that β_1 is a biased estimator for θ_m because the values do not lie on the 45^o line. However, β_1 varies near linearly with θ_m. These observations motivated an estimation procedure that matches the data moments β_k to the moments $\beta_k(\theta_m)$ implied by the simulated model under a known θ_m.

To provide evidence that the estimation procedure recovers the underlying parameter for each model, we apply the methodology on data simulated by each model under a known θ_m. For exposition purposes, we let θ_m be the same across the models, and we set it equal to four as suggested by Simonovska and Waugh (2011).

Table 1: Estimation Results With Artificial Data, True θ = 4

	Estimate of θ	"J-statistic"	β_1	β_2
BEJK	3.98 (0.01)	0.33	7.65	9.45
EK	4.02 (0.02)	0.31	5.27	6.21
Melitz - g	3.97 (0.04)	0.32	4.82	6.39
Melitz - r	3.93 (0.04)	0.32	4.03	5.26

Note: Value is the mean estimate across simulations. In each simulation there are 18 countries and 100 simulations are performed. The "J-statistic" reports the value $y(\hat{\theta})'\mathbf{W}(\hat{\theta})y(\hat{\theta})$. For the Melitz model, g stands for geometric average across varieties, r stands for random variety.

Table 1 reports the results from an overidentified estimation of θ applied to each model. Notice that the procedure successfully recovers the underlying parameter value for each model. However, the models yield different values for the moments of interest, β_1 and β_2. The Bernard, Eaton, Jensen, and Kortum (2003) model yields the highest values for the two moments, while the Melitz (2003) model yields the lowest. These differences suggest that the three models will demand different values of the elasticity parameter in order to match the moments observed in the data.

4. Results Using ICP Data

In this section, we apply our estimation strategy to real data. Our sample contains the thirty largest countries in the world (in terms of absorption). We use trade flows and production data for the year 2004 to construct trade shares. The price data used to compute aggregate price indices and proxies for trade costs come from basic-heading-level data from the 2005 round of the International Comparison Programme (ICP). The dataset has been employed in a number of empirical studies. For example, Bradford (2003) and Bradford and Lawrence (2004) use the ICP price data in order to measure the degree of fragmentation, or the level of trade barriers, among OECD countries. In addition, the authors provide an excellent description of the data-collection process. Eaton and Kortum (2002) use a similar dataset for the year 1990 in their estimation.

The ICP collects price data on goods with identical characteristics across retail locations in the participating countries during the 2003-2005 period.[5] The basic-heading level represents a narrowly-defined group of goods for which expenditure data are available. The data set contains a total of 129 basic headings, and we reduce the sample to 62 categories based on their correspondence with the trade data employed. Simonovska and Waugh (2011) provide a more detailed description of the ICP data.

The ICP provides a common list of "representative" goods whose prices are to be randomly sampled in each country over a certain period of time. A good is representative of a country if it comprises a significant share of a typical consumer's bundle there. Thus, the ICP samples the prices of a common basket of goods across countries, where the goods have been pre-selected due to their highly informative content for the purpose of international comparisons.

The models of Eaton and Kortum (2002) and Bernard, Eaton, Jensen, and Kortum (2003) give a natural common basket of goods to be priced across countries. In these models, agents in all countries consume all goods that lie within a fixed interval, $[0, 1]$. Thus, we consider this common list in the simulated models and we randomly sample the prices of its goods across countries, in order to approximate trade barriers, much like it is done in the ICP data.

In the Melitz (2003) model, different countries produce and consume different baskets of varieties of goods. We add a good to the list of common goods to be randomly priced across

[5]The ICP Methodological Handbook is available at http://go.worldbank.org/MW520NNFK0.

countries, if every country consumes at least one variety of that good. After randomly sampling 62 goods from the list, we compute the goods' prices according to the two rules discussed earlier.

4.1. Results From Overidentified Estimation

Estimation Results With ICP Data

	Estimate of θ	"J-statistic"	β_1	β_2
Data Moments	—	—	5.63	6.99
BEJK	2.81	0.06	5.59	6.99
EK	4.21	0.56	5.72	6.95
Melitz - g	3.93	0.81	5.22	7.23
Melitz - r	4.86	0.75	5.33	7.04

Note: The "J-statistic" reports the value $y(\hat{\theta})'\mathbf{W}(\hat{\theta})y(\hat{\theta})$. For the Melitz model, g stands for geometric average across varieties, r stands for random variety.

4.2. Discussion of Results

The exercise resulted in different elasticity estimates across the three models. In particular, the Bernard, Eaton, Jensen, and Kortum (2003) model yields an estimate of 2.81, the Eaton and Kortum (2002) model generates a value of 4.21, and the Melitz (2003) model suggests a range for θ between 3.93 and 4.86, depending on the price accounting methodology employed. Since, welfare is inversely related to θ, the results imply that the Bernard, Eaton, Jensen, and Kortum (2003) model generates the highest, while the Melitz (2003) model potentially yields the lowest gains from trade.

Given the stark welfare implications arising from the exercise, it is important to understand why the estimates of the parameter differ across models. Recall that, in order to estimate the trade elasticity, we compute the same moments using data generated by the three models and we minimize the distance between the moments obtained from real and artificial data. Although the three models demand identical rules in order to compute the moments of interest, the moments behave differently as functions of the elasticity parameter across models. The key difference lies in the trade-barrier approximations generated by the three models.

Eaton and Kortum's (2002) perfectly-competitive model implies that goods' relative prices across countries equal relative marginal costs of production and delivery of the lowest-cost suppliers to each market. No-arbitrage arguments bound the objects above by trade barriers.

Relative prices and costs are bounded above by trade barriers in the model of Bernard, Eaton, Jensen, and Kortum (2003) as well. For a given good, however, the relative price may lie above or below the relative cost because mark-ups are non-constant in this imperfectly-competitive environment. The crucial observation is that mark-ups are negatively related to costs. As a result, logged cost differences stochastically dominate logged price differences. Consequently, the expectations of first and second order statistics of logged cost differences are higher than their counterparts obtained from logged price differences. Trade-barrier approximations are therefore lower in Bernard, Eaton, Jensen, and Kortum's (2003) relative to Eaton and Kortum's (2002) model. The larger bias in trade-barrier estimates obtained from Bernard, Eaton, Jensen, and Kortum's (2003) model means that the model calls for a lower trade elasticity in order to match the moments in the data.

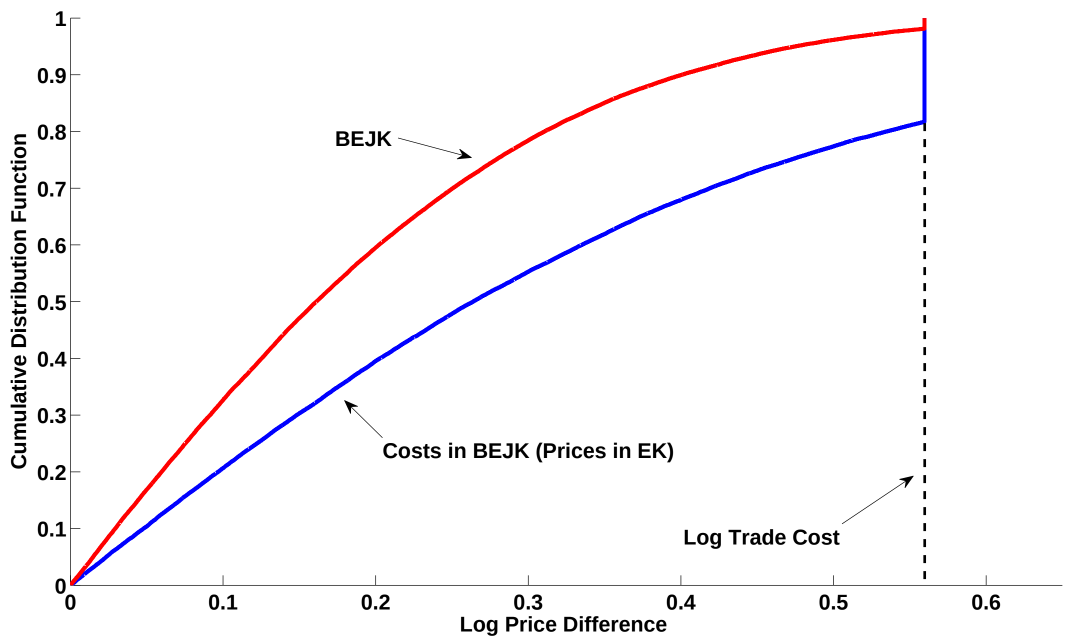

Figure 2: Distributions of logged price differences in Ricardian models

Figure 2 summarizes the above argument. It plots the CDFs of the logged price differences generated by Bernard, Eaton, Jensen, and Kortum's (2003) and Eaton and Kortum's (2002) models. Recall that price differences in Eaton and Kortum's (2002) model correspond to cost differences, which are identical in the two models. Clearly, logged price (and cost) differences are bounded above by logged trade barriers. Moreover, the distribution of logged price differences in Eaton and Kortum's (2002) model lies below the distribution generated from Bernard, Eaton, Jensen, and Kortum's (2003) model, which suggests that the former stochastically dominates the latter.

In the Ricardian models above, the maximum operator over a finite sample of prices underestimates the trade cost with positive probability and overestimates the trade cost with zero probability. Consequently, the maximum price ratio lies strictly below the true trade cost, in expectation. The argument does not apply to the monopolistically-competitive framework of Melitz (2003), where different countries supply different varieties of a good to different destinations. The relative price of a good across countries may over- or under-estimate the bilateral trade barrier, depending on the particular variety (or set thereof) used in order to compute the price ratio. Hence, in expectation, the trade-barrier approximations are higher in the Melitz (2003) model than in the Ricardian frameworks. The smaller bias in trade-barrier estimates requires a higher trade elasticity to match the moment in the data.

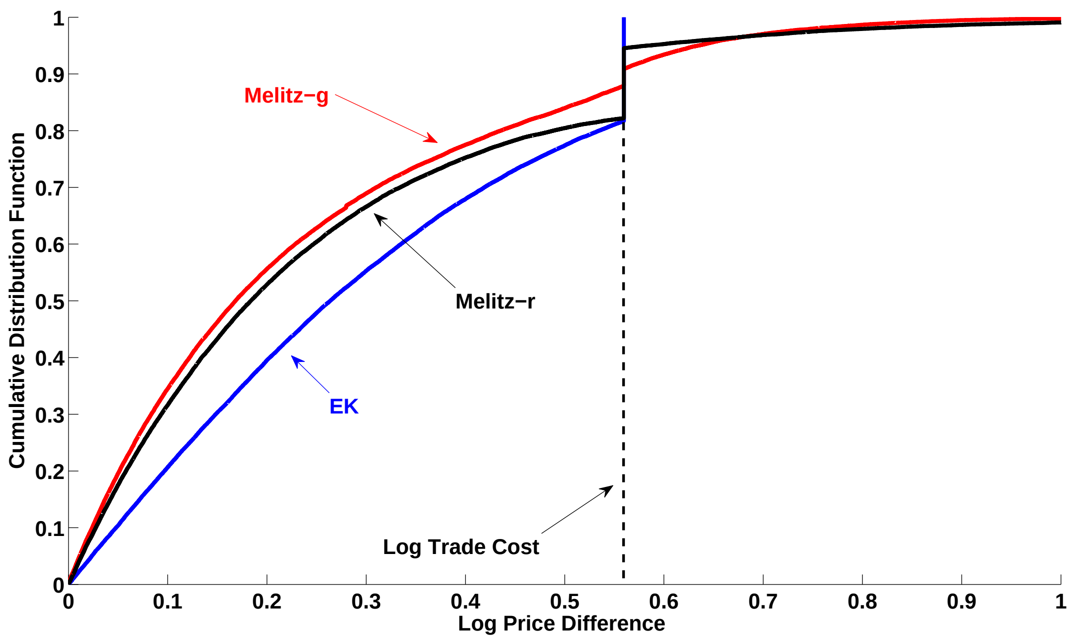

Figure 3: Distributions of logged price differences in the Melitz (2003) model

This can be seen in Figure 3, which plots the CDFs of the logged price differences generated by the two versions of the Melitz (2003) model as well as the Eaton and Kortum (2002) model. Notice that logged price differences in the Melitz (2003) model may exceed logged trade barriers because no-arbitrage arguments do not apply to varieties that originate from different sources. Hence, there is positive probability that the trade barriers will be overestimated in this model.

In addition, for the Melitz (2003) model, notice that the CDF of logged price differences computed using the geometric average lies above the CDF of logged price differences of random varieties on the domain that is bounded by the logged trade friction. Thus, if trade barriers are underestimated in the Melitz (2003) model, the bias will be larger, and the trade elasticity will be smaller, whenever the geometric average is employed. This is symptomatic of aggregation bias which arises when (geometric) averages of prices of varieties are used.

In sum, although the models share identical estimating equations for the elasticity parameter, the underlying market structure that differs across the models results in different trade elasticity estimates. Therefore, the three models yield different welfare gains from trade.

5. Conclusion

To be completed...

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A. Bernard, Eaton, Jensen, and Kortum (2003) Model

The proposition below states that logged trade barriers are bounded below by the log price difference of a good across countries.

Proposition 1 *For any good j sold in destinations n and i,* $\log p_n(j) - \log p_i(j) \leq \log \tau_{ni}$.

Proof We prove the proposition for the general case in which country a supplies good j to n and country b supplies good j to i. The triangle inequality requires that $\tau_{nb}/\tau_{ib} = \tau_{na}/\tau_{ia} = \tau_{ni}$ in order for countries n and i to buy good j from a and b, respectively. Otherwise, both countries buy good j from the same source, in which case the extra condition holds trivially. The case in which both countries share a supplier is simply a sub-case of the general proof below.

The price of good j in the two destinations is

$$p_n(j) = \min\{C_{2n}(j), \bar{m}C_{1na}(j)\}$$
$$p_i(j) = \min\{C_{2i}(j), \bar{m}C_{1ib}(j)\}$$

Since $\log$ is an increasing function,

$$\log p_n(j) = \min\{\log(C_{2n}(j)), \log(\bar{m}C_{1na}(j))\}$$
$$\log p_i(j) = \min\{\log(C_{2i}(j)), \log(\bar{m}C_{1ib}(j))\}$$

It is useful to explicitly write the pricing rules as

$$\log p_n(j) = \min\left\{\min\left[\log(C_{2na}(j)), \min_{k\neq a}\{\log(C_{1nk}(j))\}\right], \log(\bar{m}C_{1na}(j))\right\}$$
$$\log p_i(j) = \min\left\{\min\left[\log(C_{2ib}(j)), \min_{k\neq b}\{\log(C_{1ik}(j))\}\right], \log(\bar{m}C_{1ib}(j))\right\}$$

Broadly, there are four combinations of pricing rules to consider.

1. $\log(p_n(j)) = \log(\bar{m}C_{1na}(j))$ and $\log(p_i(j)) = \log(\bar{m}C_{1ib}(j))$. Then,

$$\log p_n(j) = \log(\bar{m}) + \log(w_a\tau_{na}) - \log(Z_{1a}(j))$$
$$\log p_i(j) = \log(\bar{m}) + \log(w_b\tau_{ib}) - \log(Z_{1b}(j))$$

Since n imported j from a, it must be that $\log(w_a\tau_{na}) - \log(Z_{1a}(j)) \leq \log(w_b\tau_{nb}) - \log(Z_{1b}(j))$. Then,

$$\begin{aligned}
\log p_n(j) - \log p_i(j) &= \log(\bar{m}) + \log(w_a\tau_{na}) - \log(Z_{1a}(j)) - [\log(\bar{m}) + \log(w_b\tau_{ib}) - \log(Z_{1b}(j))] \\
&= \log(w_a\tau_{na}) - \log(Z_{1a}(j)) - [\log(w_b\tau_{ib}) - \log(Z_{1b}(j))] \\
&\leq \log(w_b\tau_{nb}) - \log(Z_{1b}(j)) - [\log(w_b\tau_{ib}) - \log(Z_{1b}(j))] \\
&= \log(\tau_{nb}) - \log(\tau_{ib}) \\
&\leq \log(\tau_{ni})
\end{aligned}$$

because $\tau_{nb} \leq \tau_{ni}\tau_{ib}$ by triangle inequality.

2. $\log(p_n(j)) = \log(C_{2n}(j))$ and $\log(p_i(j)) = \log(\bar{m}C_{1ib}(j))$.

Since $\log(p_n(j)) = \log(C_{2n}(j))$, it must be that $\log(C_{2n}(j)) \leq \log(\bar{m}C_{1na}(j))$. Then, the proof follows from 1.

3. $\log(p_n(j)) = \log(C_{2n}(j))$ and $\log(p_i(j)) = \log(C_{2i}(j))$. Then there are four sub-cases.

a. $\log(p_n(j)) = \log(C_{2na}(j))$ and $\log(p_i(j)) = \log(C_{2ib}(j))$.

Since, $\log(p_n(j)) = \log(C_{2na}(j))$, it must be that $\log(C_{2na}(j)) \leq \min_{k\neq a}\{\log(C_{1nk}(j))\}$ and in particular the inequality holds for $k = b$. Moreover, $\log(C_{1ib}(j)) \leq \log(C_{2ib}(j))$ by definition. Using these two inequalities yields,

$$\begin{aligned}\log p_n(j) - \log p_i(j) &= \log(w_a\tau_{na}) - \log(Z_{2a}(j)) - [\log(w_b\tau_{ib}) - \log(Z_{2b}(j))]\\ &\leq \log(w_b\tau_{nb}) - \log(Z_{1b}(j)) - [\log(w_b\tau_{ib}) - \log(Z_{2b}(j))]\\ &\leq \log(w_b\tau_{nb}) - \log(Z_{2b}(j)) - [\log(w_b\tau_{ib}) - \log(Z_{2b}(j))]\\ &= \log(\tau_{nb}) - \log(\tau_{ib})\\ &\leq \log(\tau_{ni})\end{aligned}$$

because $\tau_{nb} \leq \tau_{ni}\tau_{ib}$ by triangle inequality.

b. $\log(p_n(j)) = \min_{k\neq a}\{\log(C_{1nk}(j))\}$ and $\log(p_i(j)) = \log(C_{2ib}(j))$.

Since $\log(p_n(j)) = \min_{k\neq a}\{\log(C_{1nk}(j))\}$, it must be that $\log(p_n(j)) \leq \log(C_{1nb}(j))$ (with equality if $k = b$). Moreover, $\log(C_{1ib}(j)) \leq \log(C_{2ib}(j))$ by definition. Using these two inequalities yields,

$$\begin{aligned}\log p_n(j) - \log p_i(j) &\leq \log(w_b\tau_{nb}) - \log(Z_{1b}(j)) - [\log(w_b\tau_{ib}) - \log(Z_{2b}(j))]\\ &\leq \log(w_b\tau_{nb}) - \log(Z_{2b}(j)) - [\log(w_b\tau_{ib}) - \log(Z_{2b}(j))]\\ &= \log(\tau_{nb}) - \log(\tau_{ib})\\ &\leq \log(\tau_{ni})\end{aligned}$$

because $\tau_{nb} \leq \tau_{ni}\tau_{ib}$ by triangle inequality.

c. $\log(p_n(j)) = \min_{k\neq a}\{\log(C_{1nk}(j))\}$ and $\log(p_i(j)) = \min_{k\neq b}\{\log(C_{1ik}(j))\}$.

It suffices to prove that the result holds when the best competitor to the producer that supplies to country n comes from k', while the best competitor to the producer that supplies to country i comes from k'', since the case in which the best competitor to both producers comes from the same country is just a sub-case.

Since k' attains the minimum for destination n, it must be that $\log(w_{k'}\tau_{nk'}) - \log(Z_{1k'}(j)) \leq \log(w_{k''}\tau_{nk''}) - \log(Z_{1k''}(j))$. Then,

$$\begin{aligned}\log p_n(j) - \log p_i(j) &= \log(w_{k'}\tau_{nk'}) - \log(Z_{1k'}(j)) - [\log(w_{k''}\tau_{ik''}) - \log(Z_{1k''}(j))]\\ &\leq \log(w_{k''}\tau_{nk''}) - \log(Z_{1k''}(j)) - [\log(w_{k''}\tau_{ik''}) - \log(Z_{1k''}(j))]\\ &= \log(\tau_{nk''}) - \log(\tau_{ik''})\\ &\leq \log(\tau_{ni})\end{aligned}$$

because $\tau_{nk''} \leq \tau_{ni}\tau_{ik''}$ by triangle inequality.

d. $\log(p_n(j)) = \log(C_{2na}(j))$ and $\log(p_i(j)) = \min_{k \neq b} \{\log(C_{1ik}(j))\}$.

Since $\log(p_n(j)) = \log(C_{2na}(j))$ it must be that $\log(C_{2na}(j)) \leq \min_{k \neq a} \{\log(C_{1nk}(j))\}$. In particular, the inequality holds for the k'' that is the best competitor to the producer who supplies to country i. Then,

$$\begin{aligned} \log p_n(j) - \log p_i(j) &\leq \log(w_{k''}\tau_{nk''}) - \log(Z_{1k''}(j)) - [\log(w_{k''}\tau_{ik''}) - \log(Z_{1k''}(j))] \\ &= \log(\tau_{nk''}) - \log(\tau_{ik''}) \\ &\leq \log(\tau_{ni}) \end{aligned}$$

because $\tau_{nk''} \leq \tau_{ni}\tau_{ik''}$ by triangle inequality.

4. $\log(p_n(j)) = \log(\bar{m}C_{1na}(j))$ and $\log(p_i(j)) = \log(C_{2i}(j))$.

Since $\log(p_n(j)) = \log(\bar{m}C_{1na}(j))$, it must be that $\log(\bar{m}C_{1na}(j)) \leq \log(C_{2n}(j))$. This reduces to case 3 above, which completes the proof.