A Trapped Factors Model of Innovation

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Abstract

By extending familiar models of growth and trade, we explain a counter-intuitive empirical finding: When firms face more import competition they undertake more innovation. In this extended model, factors of production are trapped inside a firm. As a result, an unexpected increase in import competition may encourage a firm to innovate not by raising the value of the output from innovative effort, but rather by reducing the opportunity cost of the inputs it requires. We calibrate our baseline model to the US economy before and after China's accession to the WTO. Without trapped factors, freer trade leads to a small permanent increase in the worldwide rate of growth. With trapped factors and asymmetric trade shocks, the model shows that the firms that face more import competition do relatively more innovation. It also suggests that the extra innovation induced by the combination of trapped factors and an unanticipated trade shock induces a small permanent increase in aggregate output and consumption. Because the equilibrium rate of innovation is below the social optimum, this trapped-factor effect leads to a small, positive increase in welfare. However, because this result depends on the assumption that the trade shock is unexpected, it does not provide any support for policies that try to exploit this effect. Moreover, the predicted magnitudes of the aggregate effects are sensitive to details of the innovative process about which we know very little.

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1 Introduction

The micro-empirical evidence shows that import competition stimulates innovation. The macro-theoretical challenge is to explain this finding and to quantify its importance.

Some of the micro evidence comes from case reports. Freeman and Kleiner (2005) describe how a US shoe-maker responded to rising Chinese imports by halting production of mass-market men's shoes that were no longer profitable. Rather than simply idle its factory, skilled employees, brand capital and organizational resources, the firm introduced new types of shoes for smaller niche markets. One specially designed batch of boots, run off for a local construction firm, had metal hoops in the soles that made it easier for workers to rapidly climb ladders. Making these boots took skilled engineers and R&D. The new design earned a patent. Bartel, Ichinowski and Shaw (2007) and Bugamelli, Schivardi and Zizza (2008) describe similar cases in which imports from low wage countries seem to raise innovation in firms located in a high-wage country.

The behavior reported in these case studies has also been found in less detailed statistical data on the behavior of a large numbers of firms. Bloom, Draca, and Van Reenen (2012) examine a sample of manufacturing firms, some of which were in industries that faced a much larger increase in import competition after China's accession to the WTO. According to such measures as R&D, patents, and patent citations, industries exposed to more import competition were significantly more innovative. Moreover, these industries were also more likely to adopt new technologies and better management techniques. These changes at the industry level were not merely the result of reallocation toward more innovative firms. Individual firms that faced more import competition exhibited a bigger increase in innovative effort and patenting.

The first challenge that these results pose is to explain the difference in the behavior of an individual firm before and after a trade shock. Why is it that they innovate after something bad happens? The usual presumption is that a negative shock will reduce investment, because it signals either lower expected returns or higher expected costs from more reliance on external finance. The second challenge is to explain the cross-sectional difference in the behavior between firms after the shock hits. The move to a more open trade regime could raise the return to innovation, as models of trade and growth suggest. In this case, the incentive to innovate should be higher for all firms. Once again, the usual presumption would be that the firms that face a significant loss in revenue would be no more likely, and perhaps much less likely, to take advantage of the new opportunities. Why then do the results show just the opposite, that it is the firms that face more competition from imports that undertake relatively more innovation?

2 Summary of the Results

The dynamic general equilibrium model developed here shows that adversity can indeed increase the rate of innovation if factors of production are trapped inside a firm. For the shoe company mentioned above, its workers might be trapped because they have human capital that is specific to the firm and will be lost if they move to other firms. The firm's physical capital might also be costly to uproot and sell. After the trade shock reduces the price for one of the goods that the firm had been producing, the opportunity cost goes down for inputs that are trapped within the firm. The firm does more innovation, not because of an increase in the value of a newly designed good, but rather because of a fall in the opportunity cost of the inputs used to design and produce the new good.

Our model of growth extends the lab-equipment model of growth and trade proposed in Rivera-Batiz and Romer (1991), which builds on the closed-economy model in Romer (1987). We assume a West-East model of the product cycle in which all innovation takes place in the West. (The West-East axis now seems a better way to capture trade flows between high and low wage countries than the traditional North-South axis.) The extension developed here allows not just for the extremes of autarky and free trade, but also for a continuum of intermediate degrees of trade integration indexed by a parameter ϕ, which measures the fraction of goods that are allowed to trade freely. Consistent with previous results on growth and trade, an increase in ϕ also raises the returns to innovation and increases the rate of growth of patents and the rate of growth of output, which benefits both regions.

A change in ϕ also induces a change in the terms of trade. When ϕ is low enough to impose binding restrictions on the pattern of trade, the exchange rate, measured as the cost in units of output in the West of a unit of output from the East, is less than one. As one would expect from traditional trade models, an increase in ϕ leads to a worldwide increase in the level of output. Here, the level gains are concentrated in the East. It has a cost advantage in producing any goods that are off-patent. When the trade restriction is loosened, it exports more to the West. To balance trade, the exchange rate increases toward 1, the value that would prevail under free trade. So the increase in ϕ changes the terms of trade in favor of the East.

Let $g(\phi)$ denote the steady state growth rate associated with a given level of trade integration. We calibrate the model to the US experience in the last few decades and find that the increase in the growth rate associated with a change in ϕ is modest. Increased trade with developing countries such as China could have boosted the worldwide steady state growth rate by about 0.1 percentage points, so $g(\phi') - g(\phi) \approx 0.1\%$.

For convenience, we work with an endogenous growth model in which a change in policy leads to this kind of change in the steady state growth rate. We do this because it is easier to capture the effects of policy changes if most of the analysis takes the form of a comparison of different steady state growth rates. This assumption of convenience could easily we weakened. By making an arbitrarily small change in one parameter in the model, we can convert it into a model where no positive steady state growth rate is ever feasible. By continuity, the same kind of qualitative behavior obtains for nearby values of this parameter, so the welfare and near-term growth effects of the model will be very similar whether or not the policy has permanent effects on the growth rate. The artificial discontinuity that seems to separate these two classes of models arises only if one follows the theoretically unjustified procedure of first taking the limit of the growth rate as time goes to infinity, then considering the effect of changes in the parameter.

This small parameter change converts the model into a semi-endogenous growth model of the type proposed by Jones (1995b). As Jones argues (1995a), this kind of model provides a better fit to date over horizons long enough for the stock of human capital to increase substantially. In those models, policy can have a potentially long-lasting effect on the rate of growth, which in turn has a permanent effect on future levels of the variables that grow. Nevertheless, any policy-related change to the rate of growth eventually goes away as growth converges asymptotically to zero, as it must do for any feasible policy, so the growth and welfare effects have to be captured in the type of transition dynamics that the model used here largely avoids.

To further limit the importance of any transition dynamics, we also minimize the persistence in the model. In particular, we assume that durable inputs in production last for only one period and that patents also last for only one period. With these assumptions, it takes only a few periods to converge to a new, slightly higher steady state growth rate after an unexpected change in ϕ.

Once we add trapped factors and assume that the new import competition is concentrated in a subset of firms, we can reproduce the cross-sectional results from Bloom, Draca, and Van Reenen (2012). Suppose that the increase $\phi' - \phi$ in the range of goods that are suddenly allowed as imports from the low cost East consists of goods that are made by only a subset of all firms. Let g^S (S for "Shock") denote the rate of growth of patents at firms that face this trade shock and let g^N (N for "No shock") denote the growth rate of patents at firms that face no new competition for goods that they make. When the change from ϕ to ϕ' shock is announced in period T, we find that

$$g_T^S > g_T^N > g(\phi).$$

Moreover, the difference between the two types of firms is large. In our baseline model, the number of new patents developed by a representative S firm that faces a shock jumps up to a level that is 15.1% higher than for an N firm with no shock. This cross-sectional impact on patenting rates can be seen in Figure 1, which plots for each industry the flow of new patents in the trapped factors environment, together with the patent flows which would have been obtained without trapped factors. For convenience, the pre-shock patent flows have been normalized to 1000 patents for each type of firm. The figure also shows the identical rate of growth of patents for the two types of firms when factors are fully mobile.

[Figure 1 about here.]

To indicate the effect that the trapped factors and the asymmetrical shocks have on the aggregate rate of growth in the impact period which we denote by T, let $g_T^{Trapped}$ be the aggregate rate of growth of patents when factors are trapped and the trade shock is unanticipated. Let g_T^{Mobile} denote the corresponding rate of growth when all factors are fully mobile. We find that

$$g_T^{Trapped} > g_T^{Mobile}.$$

In our calibrated baseline, this one period growth boost, while strictly positive, is quantitatively small, $g_T^{Trapped} - g_T^{Mobile} \approx 0.1\%$, about the same size as the change in the balanced growth rates, $g(\phi') - g(\phi)$, which are much more important because it holds in all future periods. This small one period trapped-factors boost in the growth rate of patents causes a permanent increase in the stock of patents. Because the decentralized rate of innovation in such models is below the social optimum, the temporary boost in the growth rate and the permanent increase in the range of intermediate inputs induced by the interaction of trapped factors and the asymmetrical trade shock causes a correspondingly small but positive increase in welfare in both the West and the East.

This kind of counter-intuitive "second-best" welfare result helps capture the essence of the positive predictions in the model, but it does not suggest new policy options such as using legislation to force higher costs on firms that lay off workers or sell off capital. The analysis here is premised on the assumption that the change in ϕ is unanticipated. If policymakers try to trap factors in firms to benefit from anticipated trade shocks, firms will reduce their input demands in response.

The magnitudes of the growth and welfare effects identified here are also sensitive to a crucial parameter in the model about which we have little prior evidence. For a calibrated version of a model like this to fit actual data, there must be some short-run decreasing returns in the technology that converts inputs into new patents. Increasing the quantities of inputs that are devoted to innovation in a period by some factor λ should not lead to an increase in the number of patents by the same factor. Following Jones and Williams (2000), we assume that patents increase instead by the factor $\lambda^{1/2}$. A simple way to understand the source of these diminishing returns is to think of innovation as a search process. If a larger team is engaged in search, the difficulties of coordinating the search effort means that there is a higher probability that different groups make redundant discoveries. With two independent discoveries of something like a long lasting light bulb, the number of new goods goes up by only one.

The key issue here is whether the challenge of avoiding redundant discoveries is entirely internal to a firm or extends across firms, as the example of the light bulb suggests. It will be largely internal if different firms naturally specialize in separate parts of search space. It will be at least partly external to an individual firm if all firms tend to search in the same parts of search space. Patent race models typically assume the extreme case of costs that are entirely external, in which case the production function for new designs exhibits a form of Marshallian external diminishing returns.

To capture the entire range of possibilities, the model allows for a parameter η that indexes the continuum of possibilities ranging from fully internal to fully external costs. The baseline specification described above, has $\eta = 1$, which implies that the costs are fully internal. In an alternative specification that allows for external costs, the magnitude of the trapped factors boost to growth should be smaller because the higher research costs (or equivalently the lower productivity of research) caused by more innovation at the shocked firms leads to an increase in the innovation cost at the no-shock firms, hence a reduction in the innovation they undertake. In a specification that allows for $\eta = 0.5$, hence a 50-50 split between internal and external costs, we find that g_T^N does go down relative to its pre-shock value $g(\phi)$, the opposite of what we found in the case illustrated in Figure 1 where $\eta = 1$. Figure 2 shows the behavior of patenting over time for the two types of firms in the $\eta = 0.5$ case. With this mixture of internal and external costs to innovative effort, the trapped factor boost to growth is still positive, so we still have the inequality

$$g_T^{Trapped} > g_T^{Mobile}.$$

However, the magnitude of the difference $g_T^{Trapped} - g_T^{Mobile}$ is about half as large as in our baseline with purely internal costs.

If we were to go to the other extreme, where $\eta = 0$ and all costs are external, the difference in the patenting rates between the shock and no-shock grows too large to fit what we observe in the data, so the combination of the micro-data and the model used here seems to suggest a point estimate for η that is more likely to be between fully internal costs and a 50-50 split than between the 50-50 split and fully external costs. Nevertheless, this kind of inference about the range of possible values for η is likely to be sensitive to modeling choices that we have not explored, so a true confidence interval that reflects both the variation in the data and the unquantified uncertainty about the structure of the model will be quite large.

We conclude that the model suggests the combination of trapped factors and asymmetric trade shocks could cause a small boost to welfare and growth, but in the absence of good evidence on the split between internal and external costs associated with redundancy in innovative effort, we can not quantify this effect with any precision.

We close with a discussion of what one could infer about aggregate effects from a microeconomic analysis like the one undertaken by Bloom, Draca, and Van Reenen (2012). As applied to data generated by our model, their approach would involve running a regression of the log of the number of new patents developed by the two types of firms on year dummies that pick up the trends and a shocked-firm dummy that picks up the difference between the S and N firms. Using data from the model, this regression would show a higher rate of patenting for the S firms in the impact period, T. With large numbers of firms, this difference could be precisely estimated, but it does not indicate anything about the aggregate effect of the shock, which will be captured in the year dummies. If there were no other variables that caused year to year fluctuations in innovation and if there were only small random fluctuations in the aggregate year to year data, the year dummies could be precisely estimated. They would have magnitudes that grow at the rate $g(\phi)$ before the shock, grow by g_T^N during the impact period, and by $g(\phi')$ after the shock. The implied rate of growth of the aggregate economy during the impact period will be an appropriately weighted average of g_T^N and the higher growth at the shocked firms implied by the dummy for the difference in the growth rates for the two types of firms.

If one could estimate g_T^N with enough precision, and if one were also confident that there were no changes in any of the other macro variables that determine the rate of innovation, the difference between Figures 1 and 2 shows that a comparison of g_T^N with $g(\phi)$ would be informative about the parameter η. As noted above, $g_T^N > g(\phi)$ is consistent with a value of η that is closer to 1 where the increasing marginal cost of innovation is internal and that $g_T^N < g(\phi)$ is consistent with a lower level of η and significant external costs. However, there is little reason to believe that all other macro variables that matter for innovation were constant during the impact period. As a result, one can have more confidence in the interpretation of the cross-sectional difference $g_T^S - g_T^N$ than the time series difference $g_T^N - g(\phi)$.

This cross-sectional effect puts useful limits on a general equilibrium model of the type developed here. In particular, it suggests that the default growth model is missing something important. Extending the model to capture these cross-sectional effects leads to new predictions about the aggregate effects. In this sense, our conclusion is more positive about the value of micro evidence in predicting aggregate behavior than the conclusions reached, for example, by Arkolakis, Klenow, Demidova and Rodríguez-Clare (2008), Atkeson and

Burstein (2010) and Arkolakis, Costinot and Rodríguez-Clare (2012). In moving to micro-data, there is no escape from the inference problems that plague macroeconomic analysis. Some of the important parameters can only be estimated from time trends, and year effects estimated from micro-data are as likely to be contaminated by changes in other variables as trends inferred from macro variables.

Because we can't rule out unmodeled macro fluctuations that affect year to year investment in innovation, the micro data and the model together cannot pin down with any precision the magnitude of the aggregate effects on output and welfare. The model suggests a positive effect, but other evidence about the extent of external costs associated with increased innovative effort will be required to be more precise.

If one is interested in the answer to a specific question such as the effect that a change in trade policy has on growth and welfare, micro evidence and macro models taken together have something to offer. The modeling exercises can suggest a set of possibilities. The micro evidence can reduce the size of this set, particularly by pushing us to extend default models that can't capture the micro behavior. Nevertheless, even in combination with parameters suggested by other macro results, the micro-evidence will not reduce the set of possibilities down to a single point that we can use to give a precise answer to questions about growth. However, as more evidence accumulates, the set of possibilities will continue to shrink.

3 Closed Economy

We introduce the basic structure of the model for a closed economy. This lets us describe the technology and highlight the key equation in the model, the one that characterizes the rate of growth of the variety of inputs, which can also be interpreted as the rate of growth of patents or new designs.

3.1 Technology

There are two types of inputs, human capital and a variety of produced intermediate inputs. At any date, these inputs can be used in three different productive activities: producing final consumption goods, producing new physical units of the intermediate inputs that will be used in production in the next period, and producing new designs or patents. We assume that the two types of inputs are used with the same factor intensities in these three activities, so we can use the simplifying device of speaking of the production first of final output, and then the allocation of final output to the production of consumption goods, intermediate inputs, or new patented designs. We can also speak of final output as the numeraire, with the understanding that it is in fact the bundle of inputs that produces one unit of final output that is actually the numeraire good.

With this convention, we can write final output Y_t in period t, as the following function of human capital H and intermediate goods $x_{jt,}$ where j is drawn from the range of intermediate inputs that have already been invented, $j \in [0, A_t]$:

$$Y_t = H^{\alpha} \int_0^{A_t} x_{jt}^{1-\alpha} dj,$$

Using the convention noted above, we can speak of firms in period t devoting a total quantity Z_t of final output to the production of new patented designs that lead to an increase from increase A_t this period to A_{t+1} next period. If we let C_t denote final consumption goods, final output is divided as follows:

$$\begin{aligned} Y_t &= C_t + K_{t+1} + Z_t \\ &= C_t + \int_0^{A_t} x_{jt+1} dj + Z_t \end{aligned}$$

The intermediate inputs are like capital that fully depreciates after one period of use.

The key equation for the dynamics of the model describes the conversion of foregone output Z_t into new patents. In period t, each intermediate goods firm f can use final goods (or more formally, the inputs that could produce final output) to discover new types of intermediate goods for use in $t+1$. Let M_{t+1} denote the aggregate measure of new goods discovered in period t, and let M_{ft+1} be the measure of these new goods produced at firm f. Here, the letter M is a mnemonic for "monopoly" because goods patented in period t will be subject to monopoly pricing in period $t+1$. Because our patents, like our capital, last for only one period, only the new designs produced in period t will be subject to monopoly pricing in period $t+1$. As noted above, these two assumptions about limited durability ensure that the economy converges quickly to a new steady state growth rate after a policy change.

As noted in the introduction, to allow for the problem that firms face in coordinating search and innovation in larger teams, we allow for a form of diminishing marginal productivity for the inputs to innovation in any given period. This diminishing marginal productivity can be internal in the sense that it depends only on the inputs devoted to innovation within the firm, or it could be external in the sense that it depends on total inputs devoted to innovation in the economy.

To capture these two extreme cases, we start first with the fully internal case, in which firm f's production of new types of goods depends on the resources devoted to innovation at firm f but not on the resources devoted to innovation in that period at other firms.

Let Z_{ft} denote these inputs into innovation at firm f. We assume that the output of new designs will also depend on the availability of all the ideas represented by the entire stock of existing innovations, A_t. Hence we can write the number of new designs at firm f as

$$M_{ft+1} = (Z_{ft})^{\rho} A_t^{1-\rho},$$

where $0 < \rho < 1$.

The exponent on A_t is the parameter mentioned in the introduction. Its value is crucial to the dynamics of the model. The choice here, $1 - \rho$ makes it possible for an economy with a fixed quantity of H to grow at a constant rate that will depend on other parameters in the model. As noted above, we could follow the suggestion in Jones(1995b) and use a smaller value for this exponent, in which case the rate of growth in the model would always converge asymptotically to zero. By continuity, the qualitative behavior of the model should be similar for values of this exponent somewhat less than $1 - \rho$.

Another way to characterize the production process for new designs is to convert the production function above to a cost function that exhibits increasing marginal costs of innovation in period t,

$$Z_{ft} = IC(M_{ft+1}^{\gamma}, A_t) = M_{ft+1}^{\gamma} A_t^{1-\gamma},$$

where $\gamma = \frac{1}{\rho} > 1$ and the function name IC is a mnemonic for Internal Costs.

The other extreme would be to assume that the costs of innovation for any one firm depend on the total amount of innovation that is taking place in the economy because independent firms could develop redundant designs. In this case, with fully external increasing costs, the aggregate production function for innovation is given by

$$M_{t+1} = (Z_t)^{\rho} A_t^{1-\rho},$$

where Z_t is the aggregate quantity of final good devoted to innovation. The corresponding aggregate cost function is

$$Z = M_{t+1}^{\gamma} A_t^{1-\gamma}.$$

In this case, the cost per new patent to an individual firm would be the average economy-wide cost of innovation

$$Z_{ft} = EC(M_{ft+1}, M_{t+1}, A_t) = \frac{M_{ft+1}}{M_{t+1}} M_{t+1}^{\gamma} A_t^{1-\gamma}.$$

where EC is a mnemonic for external costs.

To allow for intermediate degrees of internal and external costs of innovation, we nest these two versions in a cost function for firm f of the form

$$Z_{ft} = \nu \left(IC(\bullet)\right)^{\eta} \left(EC(\bullet)\right)^{1-\eta},$$

where $0 \leq \eta \leq 1$ and the inputs for the functions $IC(\bullet)$ and $EC(\bullet)$ are as given above. As η increases, the cost function exhibits a steeper marginal cost curve within each firm, with less redundancy across firms and hence weaker innovation externalities. The fully internal and fully external innovation cost benchmarks are the cases of $\eta = 1$ and $\eta = 0$, respectively.

Finally, we note that the parameter ν is a constant which we adjust so that different choices of η generate the same balanced growth rate. (See Appendix A for details.)

3.2 Preferences

A representative household in this economy consumes final good in the amount C_t each period, inelastically supplies labor input H, and has preferences over consumption streams given by

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}.$$

The representative household receives labor income, owns all the firms, and trades a one-period bond with zero net supply. As usual, if consumption grows at a constant rate $g = \frac{C_{t+1}-C_t}{C_t}$, and if r denotes the one period interest rate on loans of consumption goods, these preferences imply the usual result

$$1 + r = \frac{1}{\beta}(1+g)^{\sigma}.$$

Because the price of consumption goods is always one unit of the numeraire good, r is also the one period interest rate on loans denominated in the numeraire.

3.3 Equilibrium

To characterize the equilibrium in this closed economy, we can assume that final goods are produced by a single competitive constant returns to scale firm which demands as inputs intermediate goods and human capital. As usual, we also assume that the labor market is competitive.

It simplifies the exposition to imagine that the intermediate inputs in production are produced by N firms for some large number N. These firms design new goods and produce the intermediate inputs that the new designs make possible. Newly discovered goods are protected by a one period monopoly patent. After the patent expires, it is convenient (and harmless) to assume that the firm f that developed a good will continue to produce it. Hence, at any date t, the range of goods $[0, A_t]$ can be divided up in to N subsets $\mathbb{A}_{ft}$ of goods produced by each firm f. Roughly speaking, we would like to assume that any intermediate good j is equally likely to be assigned to any one of the N firms. To formalize this with a continuum of inputs, we assume that for any measurable subset $\mathbb{B} \subset [0, A_t]$, and any two firms f and g,the measure of the intersection $\mathbb{B} \cap \mathbb{A}_{ft}$ is equal to the measure for $\mathbb{B} \cap \mathbb{A}_{gt}$. Finally, we assume that there is a set of potential entrants, that we refer to as fast copiers, who act as a competitive fringe and force the firms that produce off-patent goods to price them at marginal cost. (This formalism is overkill for the closed economy, but becomes important when the economy is opened for trade and some goods are protected by trade restrictions.)

The equilibrium in this model take a familiar form, with perfect competition in markets other than for the goods that are protected by patents, and by monopolistic competition with a zero marginal profit condition at the margin for firms that develop new designs that will be protected by patents. (The full definition of the equilibrium for this model is given in Appendix A.)

The fundamental equation for the dynamics of the model balances the cost of developing a new patented design against the profit that can be earned from the temporary ex post monopoly that it confers. This profit can be calculated as follows. In period $t + 1$, the inverse demand for any input will be the derivative of the aggregate production function, which implies the inverse demand curve

$$p = (1 - \alpha) H^{\alpha} x^{-\alpha}.$$

The usual markup rule for a constant elasticity demand curve implies that the monopoly price p_M will be marked up by a factor $1/(1 - \alpha)$ above its marginal cost. One unit of output today can be converted into one unit of the intermediate that is available for sale tomorrow, so marginal cost in units of output tomorrow, is $(1 + r)$ and the monopoly price tomorrow can be written as

$$p_M = \frac{1 + r}{1 - \alpha}.$$

Together, these two equations imply monopoly output

$$x_M = H \left(\frac{(1 - \alpha)^2}{1 + r} \right)^{1/\alpha}. \tag{1}$$

Because profit takes the form

$$\pi = \frac{p_M x_M}{1+r} - x_M = \frac{\alpha}{1-\alpha} x_M,$$

this yields

$$\pi = \Omega\,(1+r)^{-\frac{1}{\alpha}}\,H,$$

where $\Omega = \alpha(1-\alpha)^{\frac{2-\alpha}{\alpha}}$.

One easy way to see why profit increases linearly in H is to note that the price the monopolist sets is a fixed markup over marginal cost. Hence, profit increases linearly with the quantity the monopolist sells. As in any constant returns to scale production function, at constant prices, an increase in the use of one input such as H will lead to an increase by the same factor in the quantity demanded of all complementary intermediate inputs x_j.

The zero marginal profit condition at the margin for developing new goods implies that this expression for π must be equal to the marginal cost of producing the last innovation at each firm. To express this cost, it helps to define a pseudo-growth rate for an individual firm, $g^f_{t+1} = \frac{M_{ft+1}}{A_t}$. (This is a pseudo-growth rate because we have divided by the economy-wide stock of patents rather than the firms own stock of patents. All other growth rates, including g^N and g^S, are true growth rates.) We denote the economy-wide growth rate of varieties as $g_{t+1} = \frac{M_{t+1}}{A_t}$ and note that $g_{t+1} = \sum_{f=1}^{N} g^f_{t+1}$. Differentiation of the cost function for innovation yields

$$\frac{\partial}{\partial M_{ft+1}} Z_{ft+1} = v(\eta(1-\gamma)+1)\left(g^f_{t+1}\right)^{\eta(\gamma-1)}(g_{t+1})^{(\gamma-1)(1-\eta)}$$

On a balanced growth path, g_{t+1} will be equal to a constant g, which will also be equal to the rate of growth of output and of consumption. By symmetry among the N firms, we also have that $g^f_{t+1} = \frac{1}{N}g$. As a result, the cost of a new design reduces to

$$\begin{aligned}\frac{\partial}{\partial M_{ft+1}} Z_{ft+1} &= v(\eta(1-\gamma)+1)(\frac{1}{N}g)^{\eta(\gamma-1)}(g)^{(\gamma-1)(1-\eta)} \\ &= \nu(\eta(1-\gamma)+1)N^{\eta(1-\gamma)}g^{\gamma-1}\end{aligned}$$

If we define ν so that

$$\nu(\eta(1-\gamma)+1)N^{\eta(1-\gamma)} = 1$$

the cost of a new patent reduces to $g^{\gamma-1}$. Equating this cost with the ex post profit yields

$$g^{\gamma-1} = \Omega\,(1+r)^{-\frac{1}{\alpha}}\,H$$

where $\Omega = \alpha(1-\alpha)^{\frac{2-\alpha}{\alpha}}$ is a constant of little intrinsic interest.

Finally, using the fact that in a balanced growth equilibrium, consumption, patents, and total output will all grow at the same rate g, we can substitute in the expression for the interest rate to generate the basic equation relating g and H:

Proposition 1 *Closed-Economy Balanced Growth Path*

The closed economy has a unique balanced growth path with a common constant growth rate g for varieties, output, and consumption, that satisfies the innovation optimality condition

$$g^{\gamma-1} = \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}H.$$

See Appendix A for proof.

In the closed economy, this proposition says that the cost of a patent must be equal to the appropriately discounted ex post profit that it will generate, and that this ex post profit is proportional to the stock of H. When we extend this to the open economy setting, the same kind of expression in which g is an increasing function of H will still hold except that H will be replaced by an expression that depends on both H in the West, H^* in the East, and the extent of restrictions that limit trade between the two regions.

4 Open Economy

Now assume that there are two regions or countries, West and East. Treat West as the home country so variables associated with the East are indicated with an asterisk. There are identical representative households in the East and the West. The final goods technologies of the two regions are identical, but only Western intermediate goods firms have access to the innovation technology. A firm in the East can produce any intermediate good as soon is it is off-patent. It always sell it to domestic firms.

The government in the West imposes a trade restriction which allows only a proportion ϕ of off-patent intermediate goods varieties produced in the East to be imported into the West. If we make the simplifying assumption that the goods with the lowest index values are the ones that are allowed to trade, Figure 3 describes the goods that are used in production in the West and the East. It labels these ranges from the perspective of firms in the West. The goods with the lowest index values are I (for imported) goods. In terms of production in period t,the range of the I goods is from 0 to ϕA_{t-1}. These goods are produced in the East for use in the East and produced in the East and imported into the West. Next come the R (for restricted) goods. These are produced in the West for use in the West and produced in the East for use in the East. Finally, we have the M (for monopoly) goods, which are produced in the West and used in production in both the West and the East. Hence, M_t represents the new goods developed in period $t-1$ for sale in period t; R_t represents the trade-restricted but off-patent goods available for use in production in period t; I_t represents the off patent goods that can be imported into the West for use in period t. In a small abuse of the notation, we will use the symbols I, R, and M to denote both the set of goods and its measure.

In this two economy model, we can consider a unit of final output (or equivalently the bundle of inputs that produces it) in both the East and the West. We will use output in the West as the numeraire and define an exchange rate q_t as the price in units of final output in the West of one unit of final output produced in the East. We impose trade balance in each period so there is no borrowing between West and East. Along any balanced growth path, the interest rates in the West and East will be the same, so the restriction in borrowing is binding during the short transition to the new balanced growth rate that follows a policy change. Trade balance in each period requires that the value of imports into the West, $q_t p^*_{It} I_t x_I$ is equal to the value of the goods that the West sells to the East, $p_M M_t x^*_M$.

As in the usual product cycle model, we are interested only in the case where the East has a cost advantage in producing goods that it can export. On the balanced growth path, this is equivalent to having $q_t < 1$. In our analysis, we restrict attention to the case of values of the trade parameter ϕ that are low enough to ensure that this restriction holds.

For the open economy, it helps to define a second irrelevant constant $\Psi = (1-\alpha)^{\frac{\alpha-1}{2-\alpha}}$ that is analogous to the constant $\Omega = \alpha(1-\alpha)^{\frac{2-\alpha}{\alpha}}$ for the closed economy. For any given value of the trade parameter ϕ, a straightforward extension of the analysis for the closed economy yields a two equation characterization of the balanced growth rate and the associated exchange rate:

Proposition 2 *Open-Economy Balanced Growth Path*

For low values enough of the trade parameter ϕ, the world economy follows a balanced growth path with a common, constant rate of varieties, worldwide output, and consumption in each region. The growth rate $g(\phi)$ and the exchange rate $q(\phi)$ are determined by the zero profit condition for innovation

$$g(\phi)^{\gamma-1} = \Omega\beta^{\frac{1}{\alpha}}(1+g(\phi))^{-\frac{\sigma}{\alpha}}\left(H + q(\phi)^{\frac{1}{\alpha}}H^*\right)$$

and the balanced trade condition

$$q(\phi) = \left(\frac{\phi H}{g(\phi)H^*}\right)^{\frac{\alpha}{2-\alpha}}\Psi$$

and $q(\phi) < 1$.

See Appendix A for proof.

After substitution of the second formula into the first equation, the growth rate g_ϕ can be seen to be determined by the intersection of a downward sloping innovation marginal profit curve with an upward sloping innovation marginal cost curve. The marginal profit of innovation is strictly increasing in the trade openness parameter ϕ, so the the open economy balanced growth rate is strictly increasing in ϕ. This implies that the real exchange rate q_ϕ is also strictly increasing in ϕ.

Note that the innovation optimality condition is quite similar to the one in the closed-economy version of Proposition 1. In place of H,the term $H + q_\phi^{\frac{1}{\alpha}} H^*$ now determines the extent of the demand for any input and the profit that it will generate.

An increase in ϕ, a trade liberalization, leads to an increased flow of imported I goods from West to East. Balanced trade, along with demand elasticities from final good production functions equal to $\frac{1}{\alpha} > 1$, requires an increase in the real exchange rate q_ϕ. To determine the effect of ϕ on growth, all we need to know is that Western intermediate goods firms therefore can sell any newly innovated intermediate goods to a more productive East when then has a larger effective demand for new inputs. This shifts the marginal profit curve from innovation upward and the economy moves along the upward sloping marginal cost schedule to a higher growth rate g_ϕ.

In our benchmark model, we set $\frac{H^*}{H} = 8.6$ and use $\alpha = \frac{2}{3}$ and match an aggregate growth rate of about 2% per year. (See Appendix B for details.) It follows from the equation for the exchange rate that values of ϕ less than about 0.1 will ensure that $q(\phi) < 1$. In a large economy, we would expect that a large fraction of the specialized inputs in production are nontraded and so that it takes a relatively small value of ϕ to achieve factor price equalization and to capture all the benefits from free trade.

5 Trade Shocks

The open economy analysis in the last section calculated the constant perfect foresight growth rate and interest rate associated with a constant value of the parameter ϕ. Next, we start from a balanced growth path trade at an initial level ϕ and consider the effects of an unanticipated trade shock to a more liberal trade regime with $\phi' > \phi$. To carry this exercise out, we must be more explicit about the timing of decisions relative to the announcement of the change in ϕ.

5.1 Timing with a Trade Shock

To specify the timing, it helps to return to the underlying model with three different productive activities. Recall that there is a group of consumption good producing firms that acquire H and intermediate inputs x_j and use them to produce final consumption goods using the basic production function

$$C_t = H^{\alpha} \int_0^{A_t} x_{jt}^{1-\alpha} dj.$$

There is a second group of intermediate input producing firms that demand types of inputs in the same factor proportions and that devote these inputs either to the production of new designs or to the production of physical quantities of its intermediate inputs. For a specific firm f, let inputs with an overbar denote the inputs of a particular intermediate input producing firm that are allocated to the production of new designs. To clarify the allocation decision facing such a firm, consider the special case in which all of the costs of developing new designs are internal to the firm. In this case, the number of new patents that result can be written as

$$\begin{aligned} M_{ft+1} &= (Z_{ft})^{\rho} A_t^{1-\rho} \\ &= \left(\overline{H}^{\alpha} \int_0^{A_t} \overline{x}_{jt}^{1-\alpha} dj \right)^{\rho} A_t^{1-\rho} \end{aligned}$$

In parallel, denote the inputs that firm f devotes to the production of physical units of intermediate inputs with a tilde. Firm f has developed patents in the past and will have developed new ones for use in the next period according to the expression above. Let the set of patents for firm f, both expired and still active, be denoted by $\mathbb{A}_f$. Let the subset $\mathbb{A}_f'$ denote the goods that firm f can produce at zero profit or positive profit. $\mathbb{A}_f'$ excludes goods that can now be imported at lower cost from the East. Because the firm continues to all the goods in $\mathbb{A}_f'$, its output intermediate goods that will be sold in the next period can be written as

$$\int_{\mathbb{A}_f} x_{jt+1} dj = \widetilde{H}^{\alpha} \int_0^{A_t} \widetilde{x}_{jt}^{1-\alpha} dj.$$

The total quantity of an input such as H that is controlled by firm f is the sum of $\overline{H}$, which it devotes to innovation, and $\widetilde{H}$, which it devotes to production of physical units of intermediate goods that will be available for production next period. In the same way, the total quantity of any intermediate input that it has available for production is split between these same two activities.

This means that we can think of inputs being allocated between the three productive activities in two steps. First, inputs are allocated between the consumption good producing firms and the intermediate input producing firms. Next, the intermediate input producing firms make an internal allocation decision, dividing up its inputs between innovation and the production of physical units of the intermediate inputs that will be for sale in the next period.

When ϕ is constant, a constant fraction of the off-patent goods that each intermediate input firm in the West had previously produced under trade protection are now exposed to import competition. In the aggregate, the total stock of goods that are available as imports in period t is equal to ϕ times the off-patent goods in period t, or ϕA_{t-1}. For simplicity, we assume that this process of exposure is evenly distributed across all intermediate input producing firms. For firm f, this means that if it had a measure of goods $m(\mathbb{A}'_f)$ that it produced last period, in the next period, it will have a measure $(1-\phi)m(\mathbb{A}'_f)+M_f$ where M_f is the new goods that it invents in the current period. Firms can take account of this predictable effect when they make their decisions about how much of each type of input to acquire.

In contrast, if a government mandated increase in ϕ to ϕ' takes effect in period T, there will be an additional jump in the number of goods that are subject to import competition. The measure of goods in this unexpected trade shock is $A_{t-1}(\phi'-\phi)$. We want to allow for the possibility that this range of goods is not equally distributed among all firms. Specifically, we split the set of firms into two groups, each of which have half of the intermediate input producing firms in the West. As noted above, we refer to these as the Shock and No-shock firms. We assume that all the goods that are unexpectedly exposed to competition from imports unexpected new exposure to import competition are inputs that were previously manufactured by the Shock firms.

5.1.1 Timing with Fully Mobile Inputs

With these definitions in mind, we can describe two different assumptions about the mobility of factors. Consider first the case that we refer to as "Fully Mobile Inputs" because all allocation decisions are made after the shock is announced.

1. Intermediate goods firms enter period t with completed intermediate goods.

2. The new level of the trade restriction ϕ' that will be in effect next period is announced, together with the specific goods that will no longer be protected, which thereby determines which firms are in the Shock group and which are in the No-shock group.

3. Intermediate goods firms sell their goods at the market clearing prices anticipated in $t-1$ to both domestic and foreign consumption and intermediate input producing firms. Consumption producing firms and intermediate input firms thereby acquire the inputs that they will use to produce.

4. Intermediate goods firms in the West then allocate inputs between innovation and the production of units of intermediate goods that will be available for sale in the next period.

5. The decisions that intermediate input producing firms in both the East and the West make about quantities of inputs of each type to produce are publicly observed. For off-patent goods, the competitive fringe of fast copiers in the West stands ready to enter if these quantities are too low to yield a market price equal to marginal cost for the R goods that will be sold in the West.

In this case, the trade shock of an increase from ϕ to ϕ' will be public information before any inputs are allocated to any firms. In particular, any intermediate input producing firm knows about the trade shock and knows if it is a shock firm or a no-shock firm. If the intermediate input producing firms as a group want to reduce their input demands, inputs can freely move into the production of consumption goods.

5.2 Trapped Factors Case

In the trapped factor case, we assume that the announcement of the change in ϕ comes after inputs have already been allocated to the intermediate input producing firms. This reverses the timing of steps 2 and 3 above. The new sequence is:

1. Intermediate goods firms enter period t with completed intermediate goods.

2′ Intermediate goods firms sell their goods at the market clearing prices anticipated in $t-1$ to both domestic and foreign consumption and intermediate input producing firms. Consumption producing firms and intermediate input firms thereby acquire the inputs that they will use to produce.

3′ The new level of the trade restriction ϕ' that will be in effect next period is announced, together with the specific goods that will no longer be protected, which thereby determines which firms are in the shocked group and which are in the no shock group.

2. Intermediate goods firms in the West then allocate inputs between innovation and the production of units of intermediate goods that will be available for sale in the next period.

3. The decisions that intermediate input producing firms in both the East and the West make about quantities of inputs of each type to produce are publicly observed. For off-patent goods, the competitive fringe of fast copiers in the West stands ready to enter if these quantities are too low to yield a market price equal to marginal cost for the R goods that will be sold in the West.

In this case, an intermediate input producing firm in the West could acquire inputs in period 1 under the assumption that the equilibrium will be unchanged. Then after it has acquired its inputs (and cannot relinquish them except at a prohibitive cost) it learns that ϕ will increase to ϕ' so that the incentive to produce new patented designs is higher. Everything else equal, the increase in the balanced growth rate associated with the higher value of ϕ will mean that the intermediate input producing firms as a group will face a shadow price for inputs that is higher than the market price. However, as we will see, the increase in ϕ also leads to a one period fall in the exchange rate, which raises output in the West in period 2. Faced with higher output and consumption in period 2, if they were unconstrained, consumers and firms would want to reduce the resources devoted to intermediate input producing firms and increase the resources devoted to consumption goods producing firms in period 1. This second effect will reduce the shadow value of inputs for the intermediate input producing firms. In addition, the Shock or S firms find that they face the trade shock. They have fewer goods that they can produce at zero or positive profit, which reduces their shadow price for inputs relative to the shadow price at the No-shock N firms. If they had known that they would face a shock which makes many of their lines of production unprofitable, this effect alone would have encouraged the shocked firms to acquire fewer inputs. Because this effect can lead to a net reduction in the shadow cost of inputs for a shock firm, such a firm may increase the allocation of inputs to innovation.

To calculate the full general equilibrium effects of the shock, we must take account not just of these impact effects on input demands, but also of any induced changes in interest rates and any interactions between firms because of external costs of innovation.

Before moving to the discussion of the full equilibrium, we note one final technical detail. For factors of production to be trapped in a firm, it must be the case that the firm cannot use those factors to produce a protected good that another firm had invented in the past and is still producing under trade protection. To ensure that this is so, we make the additional assumption that the cost of producing a unit of any intermediate good is substantially lower for a firm that developed the good and produced it in the past than it would be for an intermediate input producing firm that does not have this kind of experience. We then need to make sure that the incumbent producers of the protected goods do not have market power. This is where we rely on the existence of a second type of intermediate input producing firm, a "fast copier." Intermediate input producing firms can either be innovators as assumed so far or fast copiers. Fast copiers can produce intermediate inputs developed by other firms at the same cost as the other firm. All the intermediate input producing firms in the East are fast copiers. All the intermediate input producing firms that are active in the West are innovators. The pricing decisions of the innovating intermediate input producing firms are nevertheless constrained by latent fast copiers in the West who could produce at marginal cost.

6 Quantitative Exercise

Now we calibrate and perform a quantitative exercise with the model, considering the impact of trade shocks in the fully mobile and trapped factors cases. (See appendix C for more information about how we calculated these equilibria.

We take the model period as one year. We calibrate the pre-shock model economy to match long-run growth rates in the United States, and trade flows between the United States and non-OECD countries around the year 2000. Then, we consider the full range of trade shocks admitted by the model, and compare the fully mobile transition path with the transition path generated by the trapped factors mechanism. We also compute some cross-sectional measures of the rate of growth of patents at the shock and no-shock firms.

6.1 Calibration

We started by specifying the basic parameters about which we have some prior information: the human capital share, discount rate, preference curvature, the quantity of human capital in the West relative to the East, the curvature of innovation cost function, and the pre-shock trade policy parameter. These parameters are denoted by $\alpha, \beta, \sigma, H/H^*, \rho$, and ϕ. We set $\alpha = \frac{2}{3}$, $\beta = 0.98$, and $\sigma = 1$. We estimated the ratio $\frac{H^*}{H} \approx 8.6$ from international schooling data on educational attainment in the United States and non-OECD countries in the year 2000. We fix the parameter ρ to the baseline value of $\rho = 0.5$. See Appendix B for more information about the calculation of H/H^* and robustness checks with different values of ρ.

We are then left with the parameters H and ϕ to choose. Given a choice for all other parameters, we will set ϕ' to be the maximum value consistent with the restriction that the costs of production in the East are no higher than the costs in the West, which is equivalent to the restriction that the exchange rate q be less than or equal to 1.We view these as the empirically reasonable trade liberalization shocks, given the cost advantage enjoyed during this period by manufacturers in non-OECD countries such as China.

We choose the final two parameters H and ϕ to equate the model's long-run balanced growth path growth pre-shock growth rate g_ϕ and Western pre-shock imports to gross

Economy	Pre-Liberalization	Post-Liberalization
ϕ (%)	7.95	8.95
Imports to Gross Output (%)	2.1	2.325
Growth Rate (%)	2.00	2.1
Exchange Rate (North/South)	0.89	0.93
Interest Rate (%)	4.04	4.14

Table 1: Long-Run Impact of Liberalization

output ratio $\frac{I}{Y^G}$ to the values we observe in the data: a long-run growth rate of real output in the United States, 2% and the ratio of non-OECD imports to US gross output in the early 2000s, 2.1%. This procedure yields parameters $H = 0.035$ (in an arbitrary unit which makes the magnitude meaningless) and $\phi = 7.95\%$.

6.2 Comparing Equilibria with Different Amounts of Trade

Given the calibration above, we now compute the long-run impact of a trade liberalization. By trial and error, we determined that the largest possible value of ϕ' was 8.95%. To put this structural parameter change in perspective, the liberalization we consider implies a long-run movement in imports to gross output ratios in the West from 2.1% to 2.325%.

Because this table reports values for different balanced growth paths, there are no shocks, hence no difference between the fully mobile and trapped factor assumptions.

Table 1 summarizes the difference between the balanced growth equilibria with different levels of the trade parameter ϕ as determined from our calibration. Starting from the pre-shock balanced growth path growth rate of 2.0%, the trade liberalization in the long run yields a new balanced growth path growth rate of 2.1%. As noted before, the liberalization also leads to a long-run increase in the exchange rate, to balance the increased flows of I goods from East to West. This appreciation of the East makes Western goods cheaper for the East, raising the profits and effective scale for Western intermediate goods firms operating in the East. Since the returns to innovation therefore increase, so does the long-run growth rate. This increase in scale from trade and the associated increase in growth is quantitatively modest.

6.3 Transition Dynamics in the Fully Mobile Economy

Next we consider the transition dynamics of the fully mobile economy, starting from the balanced growth path associated with trade policy ϕ and allowing an unanticipated and permanent trade policy shock $\phi \rightarrow \phi'$ that is announced in period 1, to become effective in period 2. We use the same values of ϕ and ϕ' as in the comparison across balanced growth pats described above.

In the fully mobile economy, Western intermediate goods firms can immediately respond to the announcement of the trade liberalization by adjusting their input demands, along with their production and innovation decisions. In Figure 4, we plot the aggregate transition dynamics of the economy for aggregate variety growth, the real exchange rate, and output growth in the West and East. Consumption growth follows the pattern for output growth. Interest rates are implied by the pattern of consumption growth. Hence, we can focus just on growth in the number of patented goods, the exchange rate, and output in the two countries.

The full transition to the new balance growth path is complete in approximately 6 periods. Given the trade liberalization and increased flow of I goods from West to East, the exchange rate appreciates quite quickly to maintain balanced trade, leading to an associated rapid increase in the returns to innovation and hence the amount of innovation as measured by the aggregate variety growth rate.

There is slower adjustment of most other variables. In fact, immediately after the trade shock, we see a slight reduction in Western output growth rates relative to their pre-shock values. This dip in Western output growth reflects a drop in the intensive margin of I goods used in the West, since the higher exchange rates increase the cost of these goods for the Western economy. A dip in output growth is absent in the Eastern economy because the exchange rate appreciation reduces the costs of Western inputs (M goods) for the Eastern economy. The slight overshooting of variety growth in period 2 is due to the small drop in Western output growth (and hence consumption growth and interest rates), which reduces the marginal cost of innovation slightly in period 2. After period 2, variety growth and the exchange rate are close to their new balanced growth path values, while output growth, consumption growth, and interest rates smoothly adjust to their new balanced growth path values in each economy.

6.4 Transition Dynamics in the Trapped Factors Economy

We now consider the impact of the trade shock in the trapped factors economy. First, we document the cross-sectional impact of the trade shock, and then we lay out the aggregate dynamics.

6.4.1 Cross-Sectional Impact of Trapped Factors

Recall that motivated by the evidence of cross-sectional heterogeneity in the exposure of Western firms to trade liberalization shocks documented in Bloom, Draca, and Van Reenen (2012), we assume that there are two industries, each with half of the firms in the economy. As laid out earlier, one of these industries, industry S or the Shocked firms, bears the brunt of the direct effects of liberalization in that all of the liberalized R goods varieties which lose protection are in this industry. Given the calibrated values of ϕ and ϕ', this implies that the affected industry 2 loses 2.2% of its R goods production opportunities unexpectedly when the trade shock occurs, while industry N does not see a loss of R goods production opportunities.

The timing is identical to the fully mobile economy. The economy is assumed to arrive in period 1 on the balanced growth path associated with trade policy ϕ, and then it experiences an announcement in period 1 of a new, permanent trade liberalization to ϕ', effective from period 2 onwards. The rate of growth for period 1 is determined in period 0. It is equal to the balanced growth rate implied by the initial ϕ.

Since Western intermediate goods firms have already demanded final goods inputs for use in production and innovation when the trade shock occurs, they are constrained in their production and innovation decisions by adjustment costs preventing the downward adjustment of input demands. Innovating firms in the shocked industry S are no longer able to use trapped factors within the firm for the previously protected R goods production lines. In these firms, the surplus of factors leads to a 10.67% reduction in the shadow value or opportunity cost of inputs. The no shock innovating firms in industry N also experience a smaller reduction in the opportunity costs of inputs trapped within these firms, because the effect of the fall in the exchange rate in period 2, which reduces the equilibrium demand for investment in period 1, dominates the effect of the increase in ϕ, which will tend to increase investment in period 1.

[Figure 5 about here]

As Figure 5 shows, these two effects lead to a burst of innovation in period 1 that causes a higher variety growth, output. Even though the firms in the East are now allowed to sell more goods to the West, in period 2,the firms in the West will have an unusually large quantity of newly discovered goods to sell to the East, so many more that the net effect on the exchange rate in period 2 is to fall below its value in period 1.As a result, output in the West will temporarily be higher in period 2, so the growth in output and consumption in the West between periods 1 and 2 is unusually high. Since the higher interest rate increases the costliness of production for Western intermediate goods firms, the no shock innovating firms in industry 1 would like to shrink but cannot due to the adjustment constraint. The shadow value of inputs within the no shock firms in industry N therefore falls, but by a smaller amount, 2.61%, than the 10.67% fall at the shocked firms in industry S.

In period 1, Western intermediate goods firms must now allocate their over-abundant inputs between several different uses. They must determine the quantities of the remaining R goods in their portfolios to produce, as well as determine the number of new goods to innovate and produce for next period. Recall that Northern intermediate goods firms, as innovators rather than fast copiers, face prohibitively high costs of copying other firms' R goods, so they cannot shift resources into other firms' still-protected R lines. Now, intermediate goods firms have an opportunity cost of inputs lower than the interest rate in the economy, which is marginal cost of production for the competitive fringe. Therefore, the Western innovating firms will be the producer of the remaining R goods in equilibrium, and the competitive fringe will not operate. However, the marginal cost of the fringe, the interest rate, forms a ceiling for the price of the intermediate goods firms' R goods, which is chosen by them in equilibrium, pinning down R goods production in the period of the shock. This means that for the off-patent inputs that it continues to produce, both the S and N firs do not increase the quantities that the produce and sells relative to what it had expected to sell.

As a result, the lower opportunity cost of inputs within Western firms translates entirely into more inputs in innovation. The marginal cost of the inputs leads to the production of many more new M goods than would have been the case had there been no trade shock. As noted in the discussion of Figure 1, we therefore see an increase in the flow of new patents in both industries, with a relatively larger increase in innovation at firms in the shocked industry S. The patent flows for each industry, as well as industry patent flows which would have obtained for each industry in the fully mobile transition path, are plotted in Figure 1, where for convenience the pre-shock patent flow in each industry is normalized to 1000 patents. In the period of the trade shock, the larger fall in the shadow value of inputs leads to a 15.13% increase in patents in the affected industry relative to the unaffected industry. This cross-sectional variation in innovation in the face of the shock is both large and consistent with the increase documented in Bloom, Draca, and Van Reenen (2012) in patenting at Western industries directly affected by the removal of import quotas on low-cost Chinese manufacturers in the early 2000s.

In the period after the shock, however, the input adjustment constraint no longer binds and the industries again see equalized patent flows.

6.4.2 Aggregate Impact of Trapped Factors

In Figure 5 we plot the transition dynamics for aggregates in the trapped factors economy. As in Figure 4, the pre-shock and post-shock balanced growth path values of each variable are plotted for reference in red dotted and blue solid lines, respectively. The transition is again largely complete in approximately 6 periods, but the dynamics are quite different.

In particular, due to the reduction in the shadow value of inputs in both industries, and the resulting increasing in innovation, we see a large increase in patenting in period 1 and hence varieties available in period 2. The increase in the variety growth rate above its long-run new balanced growth path level puts upward pressure on output and consumption growth in the West in period 2, increasing the Western interest rate in period 2. (Again, note that although omitted, consumption growth and interest rates in both economy closely mirror the dynamics of output growth.)

As noted above, the increase in patenting in period 1 which increases the flow of M goods exports in period 2 from West to East, more than offsets the larger range of goods that the East can export to the West, so the exchange rate has to fall in period 2 to balance trade.

The Eastern economy sees an increase in output growth and consumption growth rates due to increased variety in period 2. However, the lower exchange rate, which makes M goods more expensive for the East, restrains Eastern output growth, consumption growth, and interest rates relative to the West.

After period 2, the impacts of the trapped factors constraint and the immediate increase in variety growth have essentially worked themselves through the economy. The rest of the transition resembles the fully mobile transition. In particular, the trade liberalization-implied increase in the flow of I goods from East to West requires an exchange rate appreciation in period 3. In this period and onwards, variety growth and the exchange rate are close to their new balanced growth path levels, with very small deviations as the interest rate converges in each country to its new balanced growth value.

However, the increased exchange rate, by making I goods more expensive in the West, restrains Western output and consumption growth in period 3. Since the increased exchange rate makes M goods less expensive for the East, the reverse effect is observed. This dip in output growth in the West, and increase in output growth in the East, is exactly analogous to the response to higher exchange rates observed in the fully mobile transition.

From period 3 onwards, output growth, consumption growth, and interest rates smoothly adjust to their new balanced growth path levels.

We also analyze the impact of trapped factors relative to the fully mobile benchmark by computing the welfare differences between the two cases, taking into account the full transition paths. In particular, we ask how much consumption would need to increase for households in the West and the East in the fully mobile economy in each period to make them indifferent to the trapped factors allocations. This consumption equivalent variation is approximately 0.1% in the West and the East. To understand this, note that the externalities in the innovation process through which previous ideas at one firm assist later innovation by all firms are not taken into account in the firm's innovation optimality conditions. The initial increase in variety growth due to the trapped factors mechanism helps to moderate this inefficiency and leads to a small welfare increase from our mechanism.

6.5 Internal versus External Costs

Note that our baseline results consider the case of $\eta = 1$, which corresponds to a fully internal cost structure. In this case, the increase in innovation at Shocked firms in period 1 does not directly impact the cost of innovation at the No shock firms. If we allow for external costs from redundant effort, the cost of innovation will be higher at the No shock firms, which could lead to a smaller equilibrium increase in the rate of growth. For this reason, we do a sensitivity analysis by allowing for external costs, which means letting η take on values less than 1.

Given the lack of strong empirical evidence disciplining this cost parameter, in Figure 2 we plot the microeconomic response of innovation in the case of halfway internal/halfway external costs, or $\eta = 0.5$. Importantly, the higher cost of innovation at shocked firms in the shock period amplifies the cross-sectional differences in innovation, with the shocked industry patenting 17.63% more in period 2. The larger offset of innovation at no shock firms reduces the aggregate increase in innovation from trapped factors, leading to approximately half the response in variety growth seen in the shock period in the $\eta = 1$ case. These aggregate dynamics can be seen in Figure 6, which is the analogue for this calibration of Figure 5 above. In summary, more external cost effects lead to large cross-sectional and smaller aggregate impacts from trapped factors.

We also note for completeness that when $\eta = 0$, the case of fully externalized costs of innovation, the offset becomes essentially total, with virtually no aggregate impact from trapped factors and quite large cross-sectional variation (21.17% difference in innovation in the different industries). Although we view the quite extreme cross-sectional variation in the fully external case as implausible, the parameter η is clearly crucial for the quantification of the impact of the trapped factors mechanism.

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Appendix A - Theory

There are multiple equivalent conventions which can be used in the decentralization of this economy. In the text when describing the intermediate goods firms within the West and East, we found it convenient to treat these firms as each having identical technologies but devoted to the production of consumption goods, innovation of new goods, or production of existing goods. In this structure, although we can for convenience speak of a flow Y_t of output, there is no formal "final good or final goods firm," and intermediate goods firms directly demand human capital from households and intermediate goods from other firms. However, please note that in the Appendices of the paper we have used an alternative formulation, one which is equivalent in its allocations. In the alternative Appendix formulation which is used in the definitions and proofs below, we speak of a final goods firm operating under perfect competition, which creates a physical flow of final goods output that a single class of intermediate goods firms, which are equity-financed, must direct optimally towards production and innovation in the interest of their owners. Although equivalent in terms of allocations, these formulations do involve different notation.

Definition 1 *Closed-Economy Equilibrium*

Given initial conditions A_0, x_{k0}, an equilibrium is a path of wages, interest rates, stock prices, and intermediate goods prices w_t, r_t, q_{ft}, p_{jt}, together with stock portfolio decisions, debt levels, final goods firm input demands, intermediate goods firms input demands, intermediate goods firm innovation quantities, intermediate goods dividends, aggregate innovation quantities, firm variety portfolios, and aggregate variety quantities s_{ft}, b_t, H_t^D, x_{jt}^D, x_{jt+1}^S, M_{ft+1}, d_{ft}, A_t, $A_{ft,}$ $M_{t,}$ such that

Households Optimize: Taking wages w_t, interest rates r_t, and stock prices q_{ft} as given, the representative household maximizes the present discounted value of its consumption stream by choosing period consumption C_t, debt b_{t+1}, and share purchases s_{ft}, i.e. these decisions solve

$$\max_{C_t, b_{t+1}, s_{jt}} \sum_{t=0}^{\infty} \frac{\beta^t C_t^{1-\sigma}}{1-\sigma}$$

$$b_{t+1} + C_t + \sum_{f=1}^{N} q_{ft}(s_{ft} - s_{ft-1}) \leq (1 + r_{t+1})b_t + w_t H + \sum_{f=1}^{N} d_{ft} s_{ft-1}.$$

Final Goods Firm Optimizes: Taking wages w_t and intermediate goods prices p_{kt} as given, the competitive representative final goods firm statically optimizes profits by choosing labor demand H_t^D and intermediate goods input demands x_{jt}^D, i.e. these decisions solve

$$\max_{H_t, x_{kt}} (H_t)^{\alpha} \int_0^{A_t} (x_{jt})^{1-\alpha} \, dj - w_t H_t - \int_0^{A_t} p_{jt} x_{jt} dj.$$

Intermediate Goods Firms Optimize: Taking marginal utilities m_t, perfectly competitive off-patent intermediate goods prices $p_{jt}, j \leq A_{t-1}$, and aggregate variety and innovation levels A_t, M_{t+1} as given, intermediate goods firms maximize firm value, the discounted stream of dividends, by choosing the measure of newly innovated goods M_{ft+1} to add to the existing measure of varieties A_{ft} in their portfolios, the supply of all intermediate goods for use next period x^S_{jt+1}, and the price of on-patent intermediate goods $p_{jt}, j \in (A_{t-1}, A_t]$, i.e. these quantities solve

$$\max_{p_{jt}, M_{ft+1}, x_{jt+1}} \sum_{t=0}^{\infty} m_t d_{ft}$$

$$d_{ft} + \int\limits_{A_{ft+1}} x_{jt+1} dj + Z_{ft} \leq \int\limits_{A_{ft}} p_{jt} x_{jt} dj.$$

Labor, Bond, Stock, and Intermediate Goods Markets Clear:

$$H^D_t = H,\ b_{t+1} = 0,\ s_{ft} = 1,\ x^D_{jt+1} = x^S_{jt+1}$$

Final Goods Market Clears:

$$Y_t = C_t + \int_0^{A_{t+1}} x_{jt+1} dj + \sum_{f=1}^{N} Z_{ft}$$

Innovation and Variety Consistency Conditions Hold:

$$A_{t+1} = A_t + M_{t+1},\ A_{ft+1} = A_{ft} + M_{ft+1},\ M_{t+1} = \sum_{f=1}^{N} M_{ft+1},\ A_t = \sum_{f=1}^{N} A_{ft.}$$

Definition 2 *Open-Economy Equilibrium*

Given any initial conditions $A_0, x_{k0,}\ x^*_{k0}$, along with a sequence of trade restrictions ϕ_t, an equilibrium in the open economy is a set of real exchange rates, interest rates, wages, stock prices, and intermediate goods prices q_t , r_t , $r^*_t, w_t, w^*_t, q_{ft}, q^*_{ft}$,p_{jt}, and p^*_{jt} , along with stock portfolio decisions, debt levels, final goods firm input demands, intermediate goods firms input demands, intermediate goods firm innovation quantities, intermediate goods firm portfolios, intermediate goods dividends, aggregate innovation quantities, imported variety measures, restricted variety measures, and aggregate variety quantities $s_{ft}, s^*_{ft}, b_{t+1}, b^*_{t+1}, H^D_t, H^{*D}_t, x^D_{jt}, x^{*D}_{jt}$, $x^S_{jt+1}, x^{*S}_{jt+1}, M_{ft+1}$, $A_{jt}, A^*_{ft,} d_{ft}, d^*_{ft}, M_{t,} I_t, R_t$, and A_t such that

Western Household Optimizes: Taking wages w_t, interest rates r_t, and stock prices q_{ft} as given, the representative household in the West maximizes the present discounted value of its consumption stream by choosing period consumption u_t, debt b_{t+1}, and share purchases s_{ft}, i.e. these decisions solve

$$\max_{C_t, b_{t+1}, s_{jt}} \sum_{t=0}^{\infty} \frac{\beta^t C_t^{1-\sigma}}{1-\sigma}$$

$$b_{t+1} + C_t + \sum_{f=1}^{N} q_{ft} \left(s_{ft} - s_{ft-1} \right) \leq (1 + r_{t+1}) b_t + w_t H + \sum_{f=1}^{N} d_{ft} s_{ft-1} .$$

Eastern Household Optimizes: Taking wages w_t^*, interest rates r_t^*, and stock prices q_{ft}^* as given, the representative household in the East maximizes the present discounted value of its consumption stream by choosing period consumption C_t^*, debt b_{t+1}^*, and share purchases s_{ft}^*, i.e. these decisions solve

$$\max_{C_t^*, b_{t+1}^*, s_{ft}^*} \sum_{t=0}^{\infty} \frac{\beta^t \left(C_t^*\right)^{1-\sigma}}{1-\sigma}$$

$$b_{t+1}^* + C_t^* + \sum_{f=1}^{N} q_{ft}^* \left(s_{ft}^* - s_{ft-1}^* \right) \leq (1 + r_{t+1}^*) b_t^* + w_t^* H^* + \sum_{f=1}^{N} d_{ft}^* s_{ft-1}^* .$$

Western Final Goods Firm Optimizes: Taking wages w_t and intermediate goods prices p_{jt} as given, the competitive representative final goods firm in the West statically optimizes profits by choosing labor demand H_t^D and intermediate goods input demands x_{jt}^D, i.e. these decisions solve

$$\max_{H_t, x_{jt}} (H_t)^{\alpha} \int_0^{A_t} (x_{jt})^{1-\alpha} \, dj - w_t H_t - \int_0^{A_t} p_{jt} x_{jt} \, dj.$$

Eastern Final Goods Firm Optimizes: Taking wages w_t^* and intermediate goods prices p_{jt}^* as given, the competitive representative final goods firm in the East statically optimizes profits by choosing labor demand H_t^{*D} and intermediate goods input demands x_{jt}^{D*}, i.e. these decisions solve

$$\max_{H_t^*, x_{jt}^*} (H_t^*)^{\alpha} \int_0^{A_t} \left(x_{jt}^*\right)^{1-\alpha} \, dj - w_t^* H_t^* - \int_0^{A_t} p_{jt}^* x_{jt}^* dj .$$

Western Intermediate Goods Firm Optimizes: Taking marginal utilities m_t, perfectly competitive off-patent intermediate goods prices $p_{jt}, j \leq A_{t-1}$, and aggregate variety, trade, and innovation levels A_t, R_t , and M_{t+1} as given, intermediate goods firms f in the West maximize firm value, the discounted stream of dividends, by choosing the measure of newly innovated goods M_{ft+1} to add to the existing measure of varieties A_{ft} in their portfolios, the supply of all intermediate goods in their portfolio for use next period x^S_{jt+1}, x^{*S}_{jt+1}, and the price of on-patent intermediate goods $p_{jt}, j \in (A_{t-1}, A_t]$, i.e. these quantities solve

$$\max_{p_{jt}, M_{ft+1}, x_{jt+1}, x^*_{jt+1}} \sum_{t=0}^{\infty} m_t \; d_{ft}$$

$$d_{ft} + \int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj + Z_{ft} \leq \int\limits_{A_{ft}} p_{jt} \, (x_{jt} + x^*_{jt})dj.$$

Eastern Intermediate Goods Firm Optimizes: Taking marginal utilities m^*_t and perfectly competitive off-patent intermediate goods prices $p^*_{jt}, j \leq A_{t-1}$ as given, intermediate goods firms f in the East maximize firm value, the discounted stream of dividends, by choosing the supply of all intermediate goods in their portfolios A^*_{ft} for use next period x^S_{jt+1}, x^{*S}_{jt+1}, i.e. these quantities solve

$$\max_{M_{ft+1}, x_{jt+1}, x^*_{jt+1}} \sum_{t=0}^{\infty} m^*_t \; d_{ft}$$

$$d^*_{ft} + \int\limits_{A^*_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj \leq \int\limits_{A^*_{ft}} p^*_{jt}(x_{jt} + x^*_{jt})dj.$$

Labor, Bond, Stock, and Intermediate Goods Markets Clear

$$\begin{aligned} H^D_t &= H, \; H^{*D}_t = H^*, \\ b_{t+1} &= 0, \; b^*_{t+1} = 0, \\ s_{ft} &= 1, \; s^*_{ft} = 1, \\ x^D_{jt} &= x^S_{jt}, \; x^{*D}_{jt} = x^{*S}_{jt}. \end{aligned}$$

Final Goods Markets Clear

$$Y_t = H^\alpha \int x^{1-\alpha}_{jt} dj = C_t + R_{t+1} x_{Rt+1} + M_{t+1}(x_{Mt+1} + x^*_{Mt+1}) + \sum_{f=1}^{N} Z_{ft}$$

$$Y^*_t = (H^*)^\alpha \int_0^{A_t} \left(x^*_{jt}\right)^{1-\alpha} dj = C^*_t + R_{t+1} x^*_{Rt+1} + I_{t+1}(x_{It+1} + x^*_{It+1})$$

No Arbitrage Pricing Condition Holds

$$p_{jt} = q_t p^*_{jt}$$

Trade is Balanced

$$I_t p_{It} x_{It} = M_t p_{Mt} x^*_{Mt}$$

Innovation and Variety Consistency Conditions Hold:

$$\phi_t \ (R_t + I_t) = I_t \ , I_t + R_t = A_{t-1} \ , I_t + R_t + M_t = A_t \ ,$$

$$A_{ft+1} = A_{ft} + M_{ft+1}, \ M_t = \sum_{f=1}^{N} M_{ft}, \ M_t + R_t = \sum_{f=1}^{N} A_{ft}, \ I_t + R_t = \sum_{f=1}^{N} A^*_{ft}.$$

Eastern Cost Advantage Condition Holds: Off-restriction goods are always produced in the Eastern economy only.

Although the fully mobile economy has essentially the same equilibrium concept as laid out in the previous section initially discussing the open economy, we must be more explicit about the trapped factors environment. In the trapped factors equilibrium, Western intermediate goods firms face an additional constraint due to the adjustment costs preventing them from immediately responding in their input usage to the new trade shock. Formally, they must solve the modified problem

$$\max_{p_{ft}, M_{ft+1}, x_{jt+1}, x^*_{jt+1}, X_{ft}} \sum_{t=0}^{\infty} m_t d_{ft}$$

$$d_{ft} + \int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj + Z_{ft} \leq \int_{A_{ft}} p_{jt} (x_{jt} + x^*_{jt}) dj$$

$$\int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj + Z_{ft} \geq X_{ft} \left(\phi^E_{t,t+1} \right),$$

where $X_{ft} \left(\phi^E_{t,t+1} \right)$ is the optimal input demand for period t, given expectations of the trade restriction $\phi^E_{t,t+1}$ for the next period. X_{ft} is also indexed by f and depends both upon the number of M goods that the firm plans to produce for next period, as well as the number of R goods that the firm has in its portfolio and plans to produce for the next period. Therefore, although these portfolio shares are only allocative in a period in which a trade shock occurs, we must be explicit about the structure we assume for the pre-shock portfolios of R goods held by each firm f, as well as the actual allocation of the trade shock liberalization among existing firms' measures of R goods.

We now define some additional notation. Let s_f be the share of off-patent R goods production firm f anticipates doing before the trade shock, where $\sum_{f=1}^{N} s_f = 1$. Then, let the trade shock allocate destruction of R goods production opportunities across firms so that only the proportion χ_f of R goods varieties can still be produced in each firm. As long as we have the consistency condition

$$\sum_{f=1}^{N} s_f \chi_f \left(1-\phi\right) A_t = (1-\phi') A_t,$$

an arbitrary choice of χ_f will be consistent with the trade shock $\phi \to \phi'$. In the fully external costs case, where $\eta = 0$ and firms pay the average cost of innovation in the economy for each new good, the distribution of production decisions will have no impact on aggregate allocations. However, with any internally increasing marginal costs of innovation ($\eta > 0$), the distribution of production decisions will be allocative for aggregate quantities.

We will henceforth make the assumption that $s_f = \frac{1}{N}$ for all firms, i.e. that pre-shock allocations of R goods production is uniform across firms. This assumption grows naturally out of our structure in which we assume that firms continue to be the producers of goods which they invented, even after these goods fall off-patent and become perfectly competitive.

We also will now assume that N is even, and that half of the firms in the economy are in a no shock industry, industry 1. The other half of firms in the economy, those in industry 2, experience a loss of R goods production opportunities during the trade shock with only a fixed proportion χ_2 of R goods remaining.

This framework is a rough approximation of the heterogeneity in the direct effects on firms in developed countries during the trade liberalizations of the early 2000s. Seen in this light, industries such as textiles which experienced a substantial loss of protection against manufacturers in low-cost economies such as China, can be identified with industry 2, while other industries would be represented by firms in group 1 in our environment.

We now define a trapped factors equilibrium formally.

Definition 3 *Trapped Factors Trade Shock Equilibrium*

Given any initial conditions $A_0, x_{k0,} x^*_{k0}$ and a sequence of trade restrictions

$$\phi_s = \begin{cases} \phi, s \leq t, \\ \phi', s > t \end{cases},$$

where the trade shift from ϕ to $\phi' > \phi$ is unanticipated and affects only industry 2, leaving the proportion χ_2 of R goods in in industry 2 restricted, a trapped factors equilibrium in the open economy is a set of real exchange rates, interest rates, wages, stock prices, and intermediate goods prices q_t, r_t, $r^*_t, w_t, w^*_t, q_{ft}, q^*_{ft}, p_{jt}$, and p^*_{jt}, along with stock portfolio decisions, debt levels, final goods firm input demands, intermediate goods firms input demands, intermediate goods firm innovation quantities, intermediate goods firm portfolios, intermediate goods dividends, aggregate innovation quantities, imported variety measures, restricted variety measures, and aggregate variety quantities $s_{ft}, s^*_{ft}, b_{t+1}, b^*_{t+1}, H^D_t, H^{*D}_t, x^D_{jt}, x^{*D}_{jt}$, x^S_{jt+1},x^{*S}_{jt+1}, M_{ft+1}, A_{ft}, A^*_{ft}, d_{ft}, d^*_{ft} M_t,I_t, R_t, and A_t such that

Western Household Optimizes: Taking wages w_t, interest rates r_t, and stock prices q_{ft} as given, the representative household in the West maximizes the present discounted value of its consumption stream by choosing period consumption C_t, debt b_{t+1}, and share purchases s_{ft}, i.e. these decisions solve

$$\max_{C_t, b_{t+1}, s_{ft}} \sum_{t=0}^{\infty} \frac{\beta^t C_t^{1-\sigma}}{1-\sigma}$$

$$b_{t+1} + C_t + \sum_{f=1}^{N} q_{ft}(s_{ft} - s_{ft-1}) \leq (1 + r_{t+1})b_t + w_t H + \sum_{f=1}^{N} d_{ft} s_{ft-1}.$$

Eastern Household Optimizes: Taking wages w^*_t, interest rates r^*_t, and stock prices q^*_{ft} as given, the representative household in the East maximizes the present discounted value of its consumption stream by choosing period consumption C^*_t, debt b^*_{t+1}, and share purchases s^*_{ft}, i.e. these decisions solve

$$\max_{C^*_t, b^*_{t+1}, s^*_{ft}} \sum_{t=0}^{\infty} \frac{\beta^t \left(C^*_t\right)^{1-\sigma}}{1-\sigma}$$

$$b^*_{t+1} + C^*_t + \sum_{f=1}^{N} q^*_{ft}(s^*_{ft} - s^*_{ft-1}) \leq (1 + r^*_{t+1})b^*_t + w^*_t H^* + \sum_{f=1}^{N} d^*_{ft} s^*_{ft-1}.$$

Western Final Goods Firm Optimizes: Taking wages w_t and intermediate goods prices p_{jt} as given, the competitive representative final goods firm in the West statically optimizes profits by choosing labor demand H^D_t and intermediate goods input demands x^D_{jt}, i.e. these decisions solve

$$\max_{H_t, x_{jt}} (H_t)^{\alpha} \int_0^{A_t} (x_{jt})^{1-\alpha} dj - w_t H_t - \int_0^{A_t} p_{jt} x_{jt} dj.$$

Eastern Final Goods Firm Optimizes: Taking wages w^*_t and intermediate goods prices p^*_{jt} as given, the competitive representative final goods firm in the East statically optimizes profits by choosing labor demand H^{*D}_t and intermediate goods input demands x^{D*}_{jt}, i.e. these decisions solve

$$\max_{H^*_t, x^*_{jt}} (H^*_t)^{\alpha} \int_0^{A_t} \left(x^*_{jt}\right)^{1-\alpha} dj - w^*_t H^*_t - \int_0^{A_t} p^*_{jt} x^*_{jt} dj.$$

Western Intermediate Goods Firm Optimizes: Taking marginal utilities m_t, perfectly competitive off-patent intermediate goods prices $p_{jt}, j \leq A_{t-1}$, and aggregate variety, trade, and innovation levels A_t, R_t, M_{t+1} as given intermediate goods firms in the West maximize firm value, the discounted stream of dividends, by first choosing the quantity of inputs $X_{ft}\left(\phi^E_{t,t+1}\right)$ given their expectations of trade policy next period, then choosing the measure of newly innovated goods M_{ft+1} to add to the existing measure of varieties A_{ft} in their portfolios, the supply of all intermediate goods in their portfolio for use next period x^S_{jt+1}, x^{*S}_{jt+1}, and the price of on-patent intermediate goods $p_{jt}, j \in (A_{t-1}, A_t]$, i.e. these quantities solve

$$\max_{p_{jt}, M_{ft+1}, x_{jt+1}, x^*_{jt+1}} \sum_{t=0}^{\infty} m_t d_{ft}$$

$$d_{ft} + \int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj + Z_{ft} \leq \int\limits_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt})dj$$
$$\int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj + Z_{ft} \geq X_{ft}\left(\phi^E_{t,t+1}\right),$$

where we have that

$$\phi^E_{s,s+1} = \begin{cases} \phi, & s \leq t \\ \phi', & s > t \end{cases}.$$

Eastern Intermediate Goods Firm Optimizes: Taking marginal utilities m^*_t and perfectly competitive off-patent intermediate goods prices $p^*_{jt}, j \leq A_{t-1}$ as given, intermediate goods firms in the East maximize firm value, the discounted stream of dividends, by choosing the supply of all intermediate goods in their portfolios A^*_{ft} for use next period x^S_{jt+1}, x^{*S}_{jt+1}, i.e. these quantities solve

$$\max_{M_{ft+1}, x_{jt+1}, x^*_{jt+1}} \sum_{t=0}^{\infty} m^*_t d_{ft}$$

$$d^*_{ft} + \int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj \leq \int\limits_{A_{ft}} p^*_{jt}(x_{jt} + x^*_{jt})dj.$$

Labor, Bond, Stock, and Intermediate Goods Markets Clear

$$H_t^D = H,\ H_t^{*D} = H^*,$$
$$b_{t+1} = 0,\ b_{t+1}^* = 0,$$
$$s_{ft} = 1,\ s_{ft}^* = 1,$$
$$x_{jt}^D = x_{jt}^S,\ x_{jt}^{*D} = x_{jt}^{*S}.$$

Final Goods Markets Clear:

$$Y_t = H^\alpha \int x_{jt}^{1-\alpha} dj = C_t + \int_{R_{t+1}} x_{jt+1} dj + \int_{M_{t+1}} (x_{jt+1} + x_{jt+1}^*) dj + \sum_{f=1}^{N} Z_{ft}$$

$$Y_t^* = (H^*)^\alpha \int_0^{A_t} \left(x_{jt}^*\right)^{1-\alpha} dj = C_t^* + \int_{R_{t+1}} x_{jt+1}^* dj + \int_{I_{t+1}} (x_{jt+1} + x_{jt+1}^*) dj$$

No Arbitrage Pricing Condition Holds

$$p_{kt} = q_t p_{kt}^*$$

Trade is Balanced

$$I_t p_{It} x_{It} = M_t p_{Mt} x_{Mt}^*$$

Innovation and Variety Consistency Conditions Hold:

$$\phi_t (R_t + I_t) = I_t, I_t + R_t = A_{t-1}, I_t + R_t + M_t = A_t,$$

$$A_{ft+1} = A_{ft} + M_{ft+1},\ M_t = \sum_{f=1}^{N} M_{ft},\ M_t + R_t = \sum_{f=1}^{N} A_{ft},\ I_t + R_t = \sum_{f=1}^{N} A_{ft}^*.$$

Eastern Cost Advantage Condition Holds: Off-restriction goods are always produced in the Eastern economy only.

Proof of Proposition 1: Closed Economy Balanced Growth Path

To complete the proof of Proposition 1, we need to show that the rates of growth of output, consumption, and varieties are equal on the balanced growth path.

The final goods market clearing condition is

$$C_t = H^{\alpha}\left[M_t x_{Mt}^{1-\alpha} + R_t x_{Rt}^{1-a} + I_t x_{It}^{1-\alpha}\right] - M_{t+1}x_{Mt+1} - R_{t+1}x_{Rt+1} - \sum_{f=1}^{N} Z_{ft},$$

where we note that since it is the measure of off-patent varieties, $R_t = A_{t-1}$, and the measure of innovated varieties $M_t = gA_{t-1}$.

Now, recall the assumption of balanced growth. If we define the growth rate of consumption by g_C, and note that the by symmetry the individual firm patenting ratios $g^f = \frac{g}{n}$, we can use the intermediate goods firm pricing rules to rewrite the final goods market clearing condition as

$$\frac{C_t}{A_t} = \frac{1}{1+g}H\left[(1-\alpha)^{\frac{1-\alpha}{\alpha}}\left((1-\alpha)^{\frac{1-\alpha}{\alpha}}+1\right)\beta^{\frac{1-\alpha}{\alpha}}(1+g_C)^{-\frac{\sigma}{\alpha}}\right] - g(1-\alpha)^{\frac{2}{\alpha}}\beta^{\frac{1}{\alpha}}(1+g_C)^{-\frac{\sigma}{\alpha}}H$$
$$- (1-\alpha)^{\frac{1}{\alpha}}\beta^{\frac{1}{\alpha}}(1+g_C)^{-\frac{\sigma}{\alpha}}H - N\nu\left(\frac{g}{N}\right)^{\eta(\gamma-1)+1} g^{(\gamma-1)(1-\eta)}.$$

Since $\frac{C_t}{A_t}$ is constant, we conclude that $g = g_C$, so that the innovation optimality condition reads

$$\frac{\nu(\eta(1-\gamma)+1)}{N^{\eta(\gamma-1)}}g^{\gamma-1} = \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}H.$$

This expression motivates the choice of the scaling constant

$$\nu = \frac{N^{\eta(\gamma-1)}}{\eta(1-\gamma)+1},$$

so that the balanced growth path growth rates are invariant to the number of firms or the degree of cost externalities across firms as well as the number of firms N. We obtain the balanced growth path innovation optimality condition

$$g^{\gamma-1} = \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}H.$$

The left-hand side, the marginal cost of innovation, is strictly increasing in g, is equal to 0 when $g = 0$, and limits to ∞ as $g \to \infty$. The right-hand side, the discounted monopoly profits from innovation, is strictly decreasing in g, is equal to $\Omega\beta^{\frac{1}{\alpha}}H > 0$ when $g = 0$, and limits to 0 as $g \to \infty$. We conclude that a balanced growth path equilibrium exists and is uniquely determined by the value of g which satisfies the innovation optimality condition. This completes the proof.

Proof of Proposition 2: Open Economy Balance Growth Path

The demand schedules for intermediate goods, based on the Western and Eastern final goods firms' technologies, are given by

$$x_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H p_{jt}^{-\frac{1}{\alpha}}$$
$$x_{jt}^{*} = (1-\alpha)^{\frac{1}{\alpha}} H^{*} \left(p_{jt}^{*}\right)^{-\frac{1}{\alpha}},$$

where p_{jt} and p_{jt}^{*} are the prices of intermediate good variety j in Western and Eastern output units, respectively, and $p_{jt} = q_t p_{jt}^{*}$.

The optimality conditions for the Western intermediate goods firm, combined with the Euler equations of the Western representative household for debt and equity, are given by

$$p_{Rt+1} = 1 + r_{t+1}$$
$$p_{Mt+1} = \frac{1 + r_{t+1}}{1-\alpha}$$
$$\frac{\partial}{\partial M_{ft+1}} Z_{ft+1} = \left(\frac{1}{1+r_{t+1}} p_{Mt+1} - 1\right)(x_{Mt+1} + x_{Mt+1}^{*}).$$

Differentiating the cost function and substituting in the optimal pricing rules we have that the third condition, the innovation optimality condition, is given by

$$\nu(\eta(1-\gamma)+1)(g_{t+1}^{f})^{\eta(\gamma-1)}(g_{t+1})^{(\gamma-1)(1-\eta)} = \Omega \beta^{\frac{1}{a}} (\frac{C_{t+1}}{C_t})^{-\frac{\sigma}{\alpha}} (H + q_{t+1}^{\frac{1}{\alpha}} H^{*}).$$

Now the balanced trade condition can be written

$$M_t p_{Mt} x_{Mt}^{*} = I_t p_{It} x_{It}$$
$$g_t A_{t-1} \frac{(1+r_t)}{1-\alpha} (1-\alpha)^{\frac{1}{\alpha}} H^{*} \left(\frac{(1+r_t)}{q_t(1-\alpha)}\right)^{-\frac{1}{\alpha}} = \phi A_{t-1} q_t (1+r_t^{*})(1-\alpha)^{\frac{1}{\alpha}} \left(q_t(1-\alpha)\right)^{-\frac{1}{\alpha}} H$$
$$q_t = \left(\frac{\phi H}{g_t H^{*}}\right)^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_t}{1+r_t^{*}}\right)^{\frac{1-\alpha}{2-\alpha}},$$

where $\Psi = (1-\alpha)^{\frac{\alpha-1}{2-\alpha}}$.

Now, applying the assumption of balanced growth, we immediately obtain from the Euler equations of both representative households that interest rates in the Western and Eastern economies, as well as the real exchange rate, are constant. Also, exactly as in the proof of Proposition 1, the final goods market clearing conditions for each economy, together with the assumption of balanced growth, imply that the ratios

$$\frac{C_t}{A_t}, \frac{C_t^{*}}{A_t}$$

are constant, so that we conclude that

$$(1+r) = (1+r^*) = \beta^{-1}(1+g)^{\sigma}.$$

Using this, we conclude that

$$q = \left(\frac{\phi H}{gH^*}\right)^{\frac{\alpha}{2-\alpha}} \Psi.$$

Now the innovation optimality condition can be rewritten as

$$g^{\gamma-1} = \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}(H + q^{\frac{1}{\alpha}}H^*).$$

Also, substituting the real exchange rate formula/balanced trade condition into the innovation optimality condition yields

$$g^{\gamma-1} = \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}\left(H + \left(\frac{\phi H}{gH^*}\right)^{\frac{1}{2-\alpha}} \Psi^{\frac{1}{\alpha}}H^*\right).$$

As a function of g, the marginal cost of innovation on the left-hand side is strictly increasing in g, starting at 0 and growing exponentially to ∞ as $g \to \infty$. The right-hand side, the discounted monopoly profits from sale of newly patented goods in the West and the East, is strictly decreasing in g, asymptoting to ∞ as $g \to 0$ and to 0 as $g \to \infty$. We conclude both that there exists a balanced growth path equilibrium for this economy, and that it is the unique balanced growth path growth rate. For any given fixed value of ϕ, we denote this growth rate, and the associated real exchange rate, by g_ϕ and q_ϕ. This completes the proof.

Appendix B - Parameter Values and Robustness Checks

6.5.1 Calculating the ratio of H to H^*

To calculate the ratio of H to H^*, we follow the human capital accounting approach in Hall and Jones (1999) and compute the human capital endowment in country c from the Barro and Lee (2010) data as $H_c = e^{0.101 S_c} P_c$, where S_c is the average number of years of schooling completed in the adult population above age 25, and P_c is the size of the population of the country c in 2000. We then define $H_{non-OECD} = 1.4 \sum\limits_{c \notin OECD} H_c$, where the ratio 1.4 corrects for the fact that not all non-OECD countries are represented in the Barro and Lee data. In particular 1.4 is equal to the ratio of the non-OECD to US population ratio in 2000 in the Wolfram Alpha database (with full global coverage) to the non-OECD to US population ratio in 2000 in the Barro and Lee data. Such a procedure relies on the implicit assumption that the schooling rates in countries not represented in the Barro and Lee data are similar to those with data present

6.5.2 Calculating the Trade Share Data

The real output growth rate is from the US NIPA tables, computed as the average annual real GDP growth rate from 1960-2010. Trade and gross output data was downloaded from the OECD-STAN database, and the non-OECD country to US imports to US gross output ratio was computed as an average over the years 1998-2001. The period was chosen to incorporate the average trade ratios from the period directly preceding the accession of China to the WTO in 2001, with the averaging conducted to smooth the effect on the trade ratio of the 2001 recession in the United States. All of the data and simple calculations performed in the calibration procedure are available on Nick Bloom's website: *http://www.stanford.edu/nbloom.*

6.6 Alternative Values for η

Parametrizations of the model with innovation cost externalities across firms, $\eta < 1$, lead to less than fully internalized costs of innovation at firms, as higher innovation at some firms, through externalities, increase innovation costs at other firms. This leads to an offsetting reduction in innovation at unaffected firms in the trapped factors case, reducing the impact of trapped factors relative to the fully mobile case. As noted in the text, in the fully external costs case with $\eta = 0$, the aggregate impact of trapped factors is essentially zero, with large cross-sectional variation. See Figures 7 and 8 for a summary of the microeconomic and macroeconomic impact of trapped factors for this calibration.

Alternative values for ρ

The parameter ρ indexes the curvature of the marginal innovation cost function, with higher values consistent with lower curvature. Since the roughly comparable lower bound for ρ suggested by Jones and Williams (2000) is our baseline value of 0.5, we consider the impact of an increase in ρ to 0.6 from its baseline level of 0.5. As can be seen in Figures 9 and 10, our central conclusions are largely unchanged from the baseline calibration presented in the text.

Appendix C - Solution Technique and Equilibrium Conditions

Please find both replication data files for the calibration exercise, as well as code to duplicate all of the quantitative results in the paper, on Nicholas Bloom's website at *http://www.stanford.edu/nbloom/.*

We solve each of the systems of nonlinear equations laid out below using particle swarm optimization as implemented in *R*. This is a standard global nonlinear optimization technique, and all solutions are computed with a summed squared percentage error across all equations of less than 10^{-7}.

Balanced Growth Path

As documented in the proof of Proposition 2, the balanced growth path growth rate g_ϕ of the open economy given trade restriction ϕ is fully characterized by the equilibrium innovation optimality condition

$$g^{\gamma-1} = \Omega\beta^{\frac{1}{a}}(1+g)^{-\frac{\sigma}{\alpha}}\left(H + \left(\frac{\phi H}{gH^*}\right)^{\frac{1}{2-\alpha}}\Psi^{\frac{1}{\alpha}}H^*\right).$$

All other long-run quantities, in particular the interest rates and exchange rate, are direct functions of this balanced growth path growth rate through the Euler equations and balanced trade condition

$$(1+r_\phi) = (1+r^*_\phi) = \beta^{-1}(1+g_\phi)^\sigma$$

$$q = \left(\frac{\phi H}{gH^*}\right)^{\frac{\alpha}{2-\alpha}}\Psi.$$

Fully Mobile Transition Dynamics

To compute the transition dynamics of the fully mobile model in response to a trade shock in period 1, starting from the balanced growth path associated with trade restriction ϕ, we first pick a horizon T. We also normalize $A_1 = 1$. Then, we assume that the model has converged to the balanced growth path associated with ϕ' by period $T+1$. This structure requires that we solve for $3(T-1)$ prices, $\{q_t, r_t, r^*_t\}_{t=2}^T$. These $3(T-1)$ prices are pinned down by $3(T-1)$ equations: the balanced trade condition, the Western Euler equation, and the Eastern Euler equation, in periods $2, ..., T$. These equations are given by

$$q_t = \left(\frac{\phi H}{g_t H^*}\right)^{\frac{\alpha}{2-\alpha}}\Psi\left(\frac{1+r_t}{1+r^*_t}\right)^{\frac{1-\alpha}{2-\alpha}},$$

$$\left(\frac{C_{t+1}}{C_t}\right)^\sigma = \beta(1+r_{t+1}),$$

$$\left(\frac{C^*_{t+1}}{C^*_t}\right)^\sigma = \beta(1+r^*_{t+1}).$$

We note that all allocations in the transition path are a function of these three prices. Intermediate goods prices follow the monopoly markup or competitive pricing conditions

$$p_{Mt} = \frac{1+r_t}{1-\alpha}, p_{Rt} = (1+r_t), p_{It} = q_t(1+r^*_t)$$

$$p^*_{Mt} = q_t^{-1}\frac{1+r_t}{1-\alpha}, p^*_{Rt} = (1+r^*_t), p^*_{It} = (1+r^*_t).$$

The final goods firms demand schedules then yield

$$x_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H p_{jt}^{-\frac{1}{\alpha}}$$
$$x^*_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H^* (p^*_{jt})^{-\frac{1}{\alpha}},$$

The first-order condition for innovation at Western intermediate goods firms, together with symmetry across firms and the equilibrium price and quantity decisions laid out above, yields the innovation optimality conditions

$$g_{t+1}^{\gamma-1} = \Omega(1+r_{t+1})^{-\frac{1}{\alpha}} \left(H + q_{t+1}^{\frac{1}{\alpha}} H^*\right),$$

which uniquely pin down the variety growth rate g_{t+1} as a function of exchange rates and interest rates. Given our characterization of g_t as a function of prices, it only remains to pin down C_t and C^*_t as a function of prices.

But this is easily accomplished by noting that

$$u_t + M_{t+1}(x_{Mt+1} + x^*_{Mt+1}) + R_{t+1}x_{Rt+1} + Z_t = Y_t$$

$$Y_t = H^\alpha \left[M_t x_{Mt}^{1-\alpha} + R_t x_{Rt}^{1-\alpha} + I_t x_{It}^{1-\alpha}\right]$$

$$Z_t = \sum_{f=1}^{N} Z_{ft} = \frac{g_{t+1}^{\gamma}}{\eta(\gamma-1)+1} A_t$$

$$C^*_t + I_{t+1}(x_{It+1} + x^*_{It+1}) + R_{t+1}x^*_{Rt+1} = Y^*_t$$

$$Y^*_t = (H^*)^\alpha \left[M_t (x^*_{Mt})^{1-\alpha} + R_t (x^*_{Rt})^{1-\alpha} + I_t (x^*_{It})^{1-\alpha}\right]$$

$$A_{t+1} = (1+g_{t+1})A_t$$

$$M_{t+1} = g_t A_t$$

$$R_{t+1} = (1-\phi_{t+1})A_t$$

$$I_{t+1} = \phi_{t+1} A_t.$$

Since all allocations in this economy are therefore a function of the $3(T-1)$ prices, we can construct the errors in $3(T-1)$ equations above given any input sequence of prices. The percentage squared errors of this system of equation are minimized using particle swarm optimization.

After solving for the transition path price paths, we check to see if the cost advantage for I goods production is maintained by the East, justifying our M, R, I goods partitioning. This is equivalent to checking that, for each period

$$(1 + r_t^*) q_t \leq (1 + r_t).$$

In the baseline results shown in Section 5, we choose $T = 7$.

Trapped Factors Transition Dynamics

The equilibrium conditions which we must solve to compute the transition dynamics for the trapped factors model are identical to those in the fully mobile economy, for period $3, ..., T$. There are, however, differences in the equilibrium conditions in the period of the shock. In particular, there is heterogeneity in the response of the affected and unaffected industries to the shock, and instead of solving for simply the $3(T-1)$ prices $\{q_t, r_t, r_t^*\}_{t=2}^T$ as in the fully mobile case, we must solve for these prices and the four additional variables $\{g_2^1, g_2^2, \mu^1, \mu^2\}$. These variables are patenting rates and shadow values of inputs within Western firms in the unaffected industry (1) and the affected industry (2). Therefore, we must pin down $3(T-1)+4$ quantities, which we do with $3(T-1)+4$ equations:

$$q_2 = \left[\frac{\phi' H}{H \left[\left(\frac{n}{2}\right) (\mu^1)^{\frac{\alpha-1}{\alpha}} g_2^1 + \left(\frac{n}{2}\right) (\mu^2)^{\frac{\alpha-1}{\alpha}} g_2^2 \right]} \right]^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1 + r_2}{1 + r_2^*} \right)^{\frac{1-\alpha}{2-\alpha}}$$

$$q_t = \left(\frac{\phi' H}{g_t H^*} \right)^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1 + r_t}{1 + r_t^*} \right)^{\frac{1-\alpha}{2-\alpha}}, 3, ..., T$$

$$\left(\frac{C_{t+1}}{C_t} \right)^{\sigma} = \beta(1 + r_{t+1}), t = 2, ..., T$$

$$\left(\frac{C_{t+1}^*}{C_t^*} \right)^{\sigma} = \beta(1 + r_{t+1}^*), t = 2, ..., T$$

$$(N g_2^1)^{\eta(\gamma-1)} (g_2)^{(\gamma-1)(1-\eta)} = \Omega (1 + r_2)^{-\frac{1}{\alpha}} (\mu^1)^{-\frac{1}{\alpha}} (H + q_2^{\frac{1}{\alpha}} H^*)$$

$$(Ng_2^2)^{\eta(\gamma-1)}(g_2)^{(\gamma-1)(1-\eta)} = \Omega(1+r_2)^{-\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(H + q_2^{\frac{1}{\alpha}}H^*)$$

$$\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(1+r_\phi)^{-\frac{1}{\alpha}}H + \frac{1}{N}\frac{g_\phi^\gamma}{\eta(\gamma-1)+1} + \frac{g_\phi}{N}(1-\alpha)^{\frac{2}{\alpha}}(1+r_\phi)^{-\frac{1}{\alpha}}(H + q_\phi^{\frac{1}{\alpha}}H^*)$$

$$= \frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}}H + \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1}\left(g_2^1\right)^{\eta(\gamma-1)+1}(g_2)^{(\gamma-1)(1-\eta)}$$

$$+g_2^1(1-\alpha)^{\frac{2}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(H + q_2^{\frac{1}{\alpha}}H^*)$$

$$\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(1+r_\phi)^{-\frac{1}{\alpha}}H + \frac{1}{N}\frac{g_\phi^\gamma}{\eta(\gamma-1)+1} + \frac{g_\phi}{N}(1-\alpha)^{\frac{2}{\alpha}}(1+r_\phi)^{-\frac{1}{\alpha}}(H + q_\phi^{\frac{1}{\alpha}}H^*)$$

$$= \frac{1}{N}\chi_2(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}}H + \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1}\left(g_2^2\right)^{\eta(\gamma-1)+1}(g_2)^{(\gamma-1)(1-\eta)}$$

$$+g_2^2(1-\alpha)^{\frac{2}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(H + q_2^{\frac{1}{\alpha}}H^*).$$

The first $3(T-1)$ equations are simply the balanced trade and Euler equations for the Western and Eastern households in periods $2, ..., T$. The balanced trade condition must be modified in period 2 to reflect the fact that flows of M goods from West to East come from both industry 1 and industry 2, with different prices and quantities for each. The final four equations represent the innovation optimality conditions for firms in industry 1 and industry 2, as well as the trapped factors constraints for firms in each industry.

The innovation optimality conditions are simply the first-order conditions of firms with respect to the mass of new varieties to be innovated in period 1 for use in period 2. Note that we are defining $\mu^1 = 1 - \lambda^1$ and $\mu^2 = 1 - \lambda^2$, where $m_1\lambda^1$ and $m_1\lambda^2$ are the multipliers on the trapped factors input constraints in the optimization problem for Western intermediate goods firms in period 1. A fall in μ below 1 represents a fall in the shadow value of inputs for an intermediate goods firm. Also, if M_{f2} is the number of new patents innovated by a firm in industry f in period 1 for use in period 2, we are following the conventions $g_2^f = \frac{M_{f2}}{A_1}$, and imposing the consistency condition

$$g_2 = \left(\frac{N}{2}\right)(g_2^1 + g_2^2).$$

The trapped factors constraints are simply the input demands for R goods production and M goods innovation and production expenditure pre-shock (left hand side) and post-shock (right hand side). The input constraints differ across industries because the R goods available in the post-shock period in industry 2 for production are reduced by the factor χ_2, where χ_2 satisfies

$$\frac{1+\chi_2}{2} = \frac{1-\phi'}{1-\phi},$$

which is the consistency condition discussed in the text. Also, the right-hand side on the trapped factors constraints take into account the following optimal pricing rules in the period of the shock:

$$p^1_{M2} = \mu^1 \frac{1+r_2}{1-\alpha}, p^1_{R2} = (1+r_2),$$
$$p^2_{M2} = \mu^2 \frac{1+r_2}{1-\alpha}, p^2_{R2} = (1+r_2).$$

The demand conditions are identical to those laid out in the fully mobile section. Intermediate goods firm innovation costs on the right hand side of the trapped factors constraint are given by

$$Z^1_2 = \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1} \left(g^1_2\right)^{\eta(\gamma-1)+1} (g_2)^{(\gamma-1)(1-\eta)}$$

$$Z^2_2 = \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1} \left(g^2_2\right)^{\eta(\gamma-1)+1} (g_2)^{(\gamma-1)(1-\eta)},$$

which is a direct application of the definition of the innovation cost function. All of the other quantities needed for construction of the Euler equation errors and balanced trade conditions are identical to those in the fully mobile economy, with the exception of the resource constraints in the West and East in periods 1 and 2 which must be modified to read

$$Y_1 = C_1 + \left(\frac{N}{2}\right) g^1_2 A_1 (x^1_{M2} + x^{*1}_{M2}) + \left(\frac{N}{2}\right) g^2_2 A_1 (x^2_{M2} + x^{*2}_{M2}) + \left(\frac{N}{2}\right) \frac{1-\phi}{2} A_1 x^1_{R2} + \left(\frac{N}{2}\right) \frac{(1-\phi)\chi_2}{2} A_1 x^2_{R2} + Z^1_2 +$$

$$Y^*_1 = C^*_1 + (1-\phi')A_1 x^*_{R2} + \phi' A_1 (x^*_{I2} + x_{I2})$$

$$Y_2 = H^\alpha \left[\left(\frac{N}{2}\right) g^1_2 A_1 (x^1_{M2})^{1-\alpha} + \left(\frac{N}{2}\right) g^2_2 A_1 (x^2_{M2})^{1-\alpha} + \left(\frac{N}{2}\right) \frac{1-\phi}{2} A_1 (x^1_{R2})^{1-\alpha} + \left(\frac{N}{2}\right) \frac{(1-\phi)\chi_2}{2} A_1 (x^2_{R2})^{1-}\right.$$

$$+\phi' A_1 x_{I2}^{1-\alpha}]$$

$$Y_2^* = (H^*)^\alpha \left[\left(\frac{N}{2}\right) g_2^1 A_1 (x_{M2}^{*1})^{1-\alpha} + \left(\frac{N}{2}\right) g_2^2 A_1 (x_{M2}^{*2})^{1-\alpha} + (1-\phi') A_1 (x_{R2}^*)^{1-\alpha} + \phi' A_1 (x_{I2}^*)^{1-\alpha} \right].$$

After computing the transition path in the above manner, we must verify that $\mu^1, \mu^2 < 1$, justifying our imposition of the trapped factors inequality constraint as an equality constraint. We must also check the Eastern cost dominance condition for I goods in each period, i.e.

$$\min(\mu^1, \mu^2)(1 + r_2) \geq q_2(1 + r_2^*),$$

$$(1 + r_t) \geq q_t(1 + r_t^*), t = 3, .., T,$$

$$q_1, q_{T+1} \leq 1.$$

Welfare Calculations

We compute the consumption equivalent variation of trapped factors relative to the fully mobile economy by first computing

$$W^{TF} = \sum_{t=1}^{\infty} \beta^{t-1} \log(C_t^{TF}), W^{*TF} = \sum_{t=1}^{\infty} \beta^{t-1} \log(C_t^{*TF})$$

$$W^{FM} = \sum_{t=1}^{\infty} \beta^{t-1} \log(C_t^{FM}), W^{*FM} = \sum_{t=1}^{\infty} \beta^{t-1} \log(C_t^{*FM}),$$

where the consumption allocations on the computed transition path from $1, ..., T$ are directly computed and consumption is assumed to grow at the rate $g_{\phi'}$ for all economies from period $T+1$ onwards. Then, we solve for x and x^*,

$$\sum_{t=1}^{\infty} \beta^{t-1} \log((1+x) C_t^{FM}) = \sum_{t=1}^{\infty} \beta^{t-1} \log(C_t^{TF}),$$

$$\sum_{t=1}^{\infty} \beta^{t-1} \log((1+x^*) C_t^{*FM}) = \sum_{t=1}^{\infty} \beta^{t-1} \log(C_t^{*TF}).$$

The welfare numbers reported in the text are $100x$ and $100x^*$.

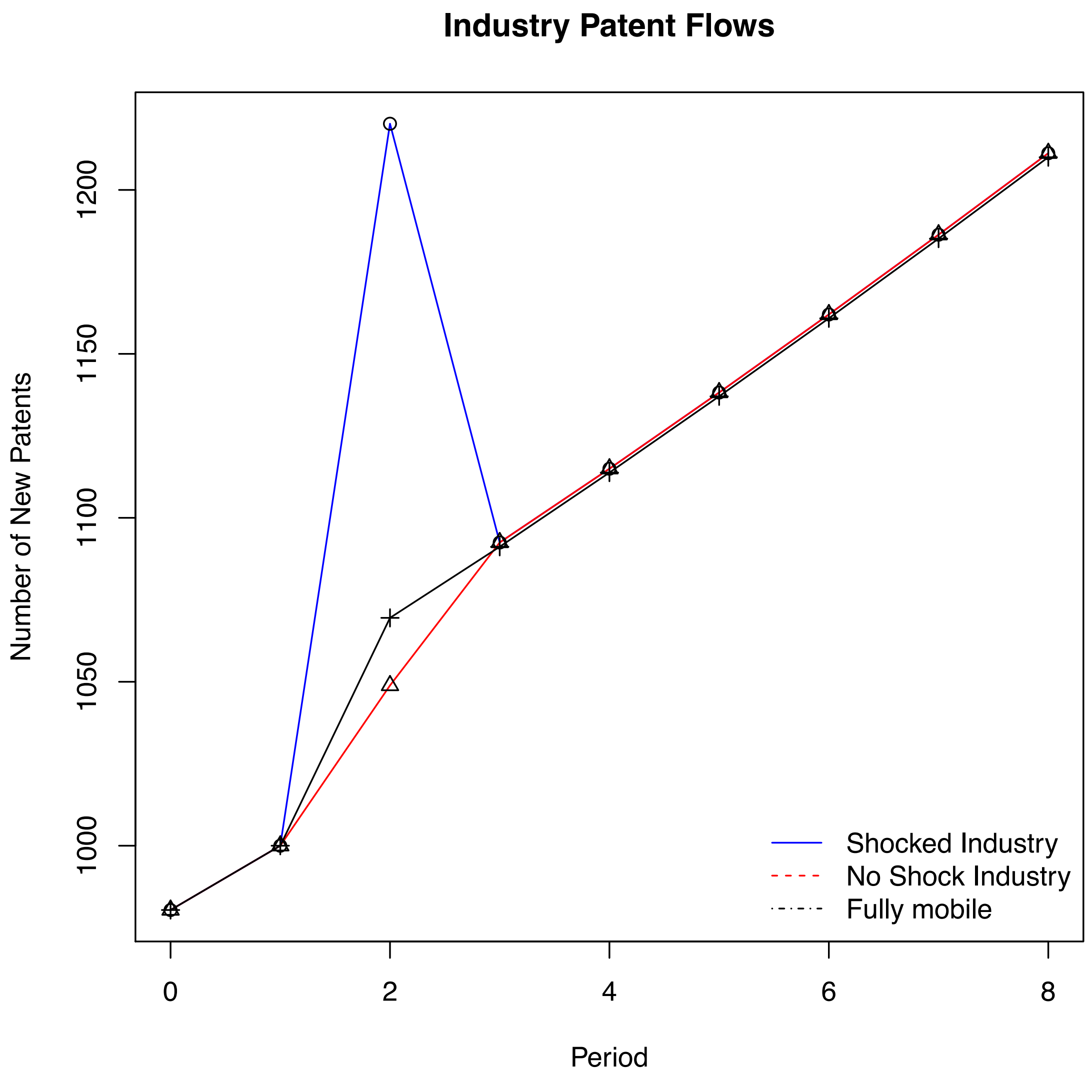

Figure 1: Cross-Sectional Impact of Trapped Factors

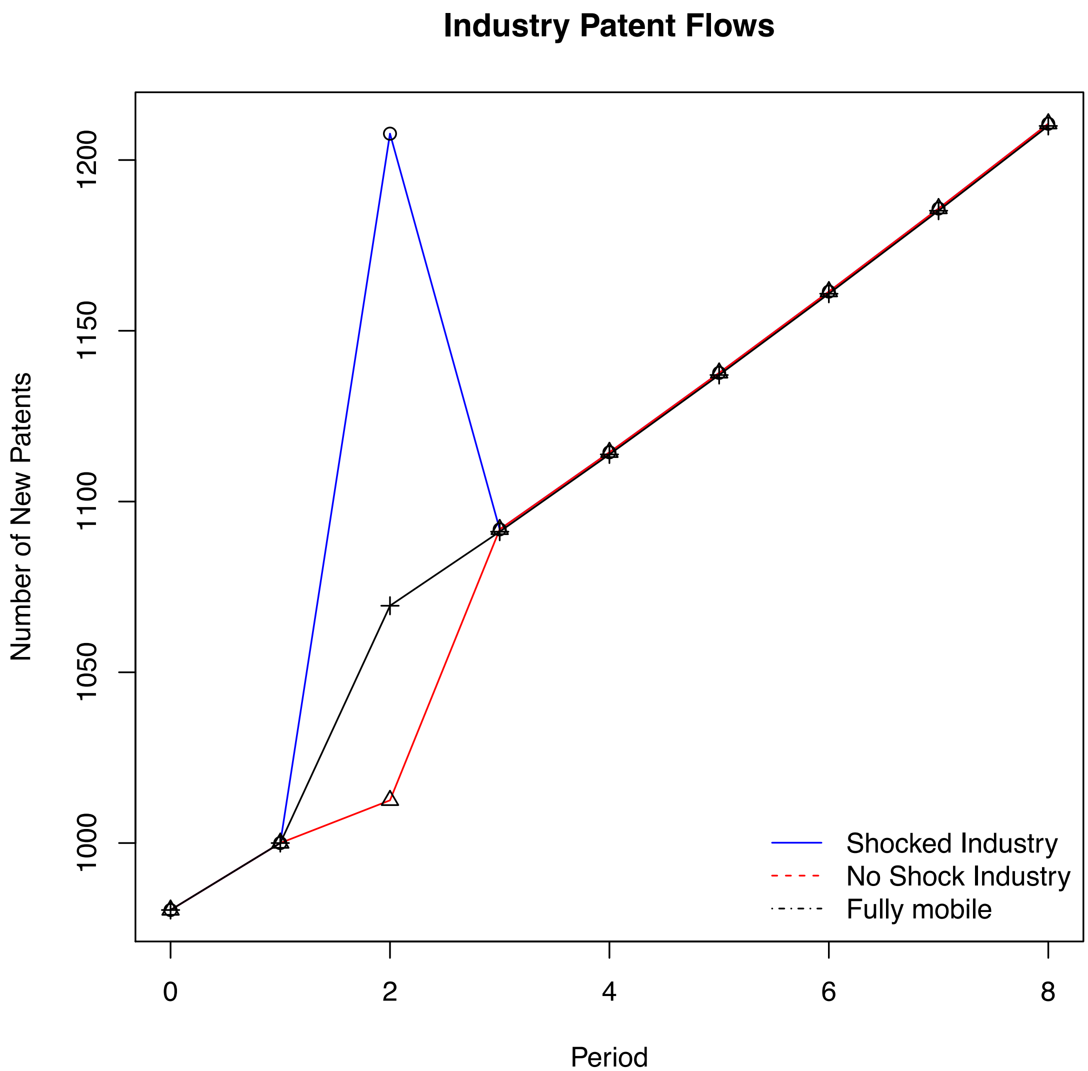

Figure 2: Cross-Sectional Impact of Trapped Factors, $\eta = 0.5$

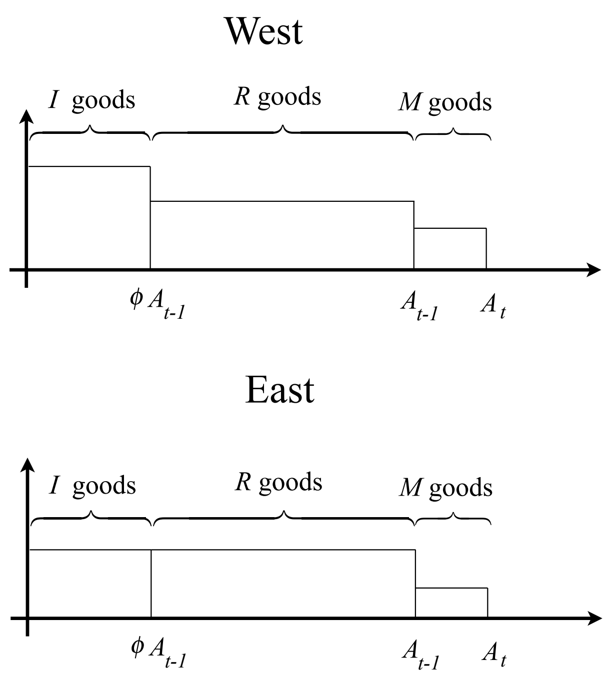

Figure 3: Product Cycle

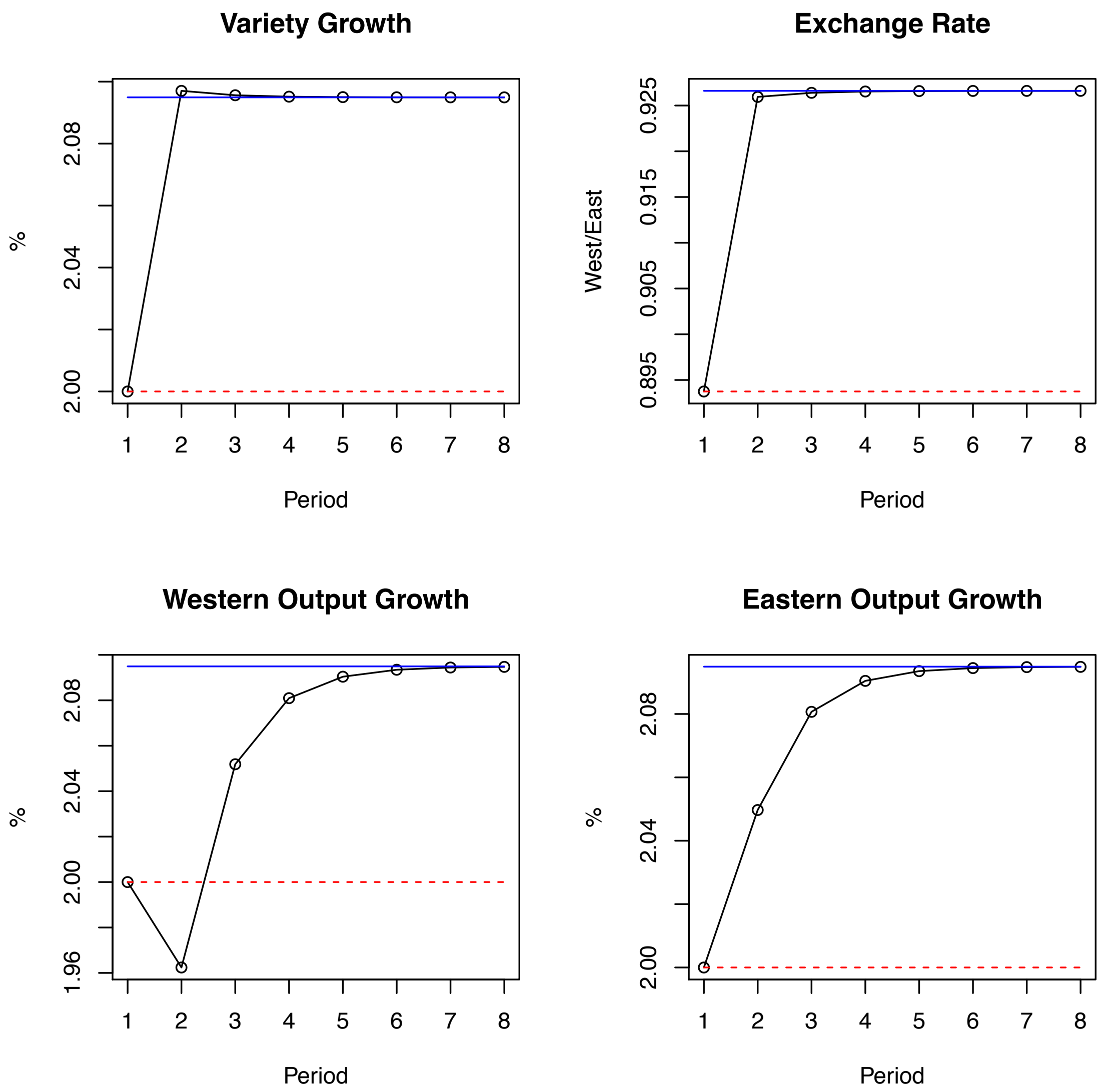

Figure 4: Fully Mobile Transition Dynamics

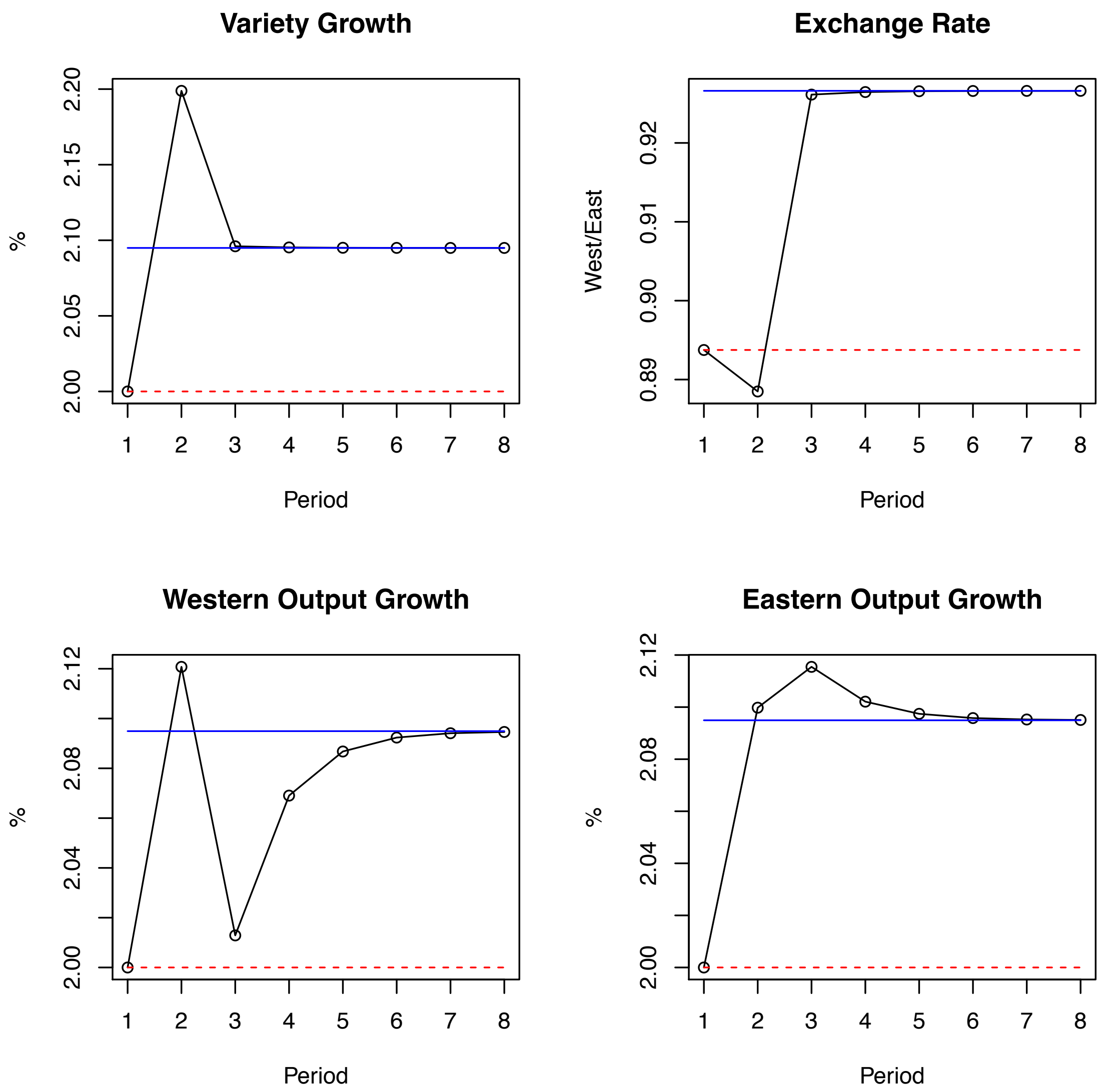

Figure 5: Trapped Factors Transition Dynamics

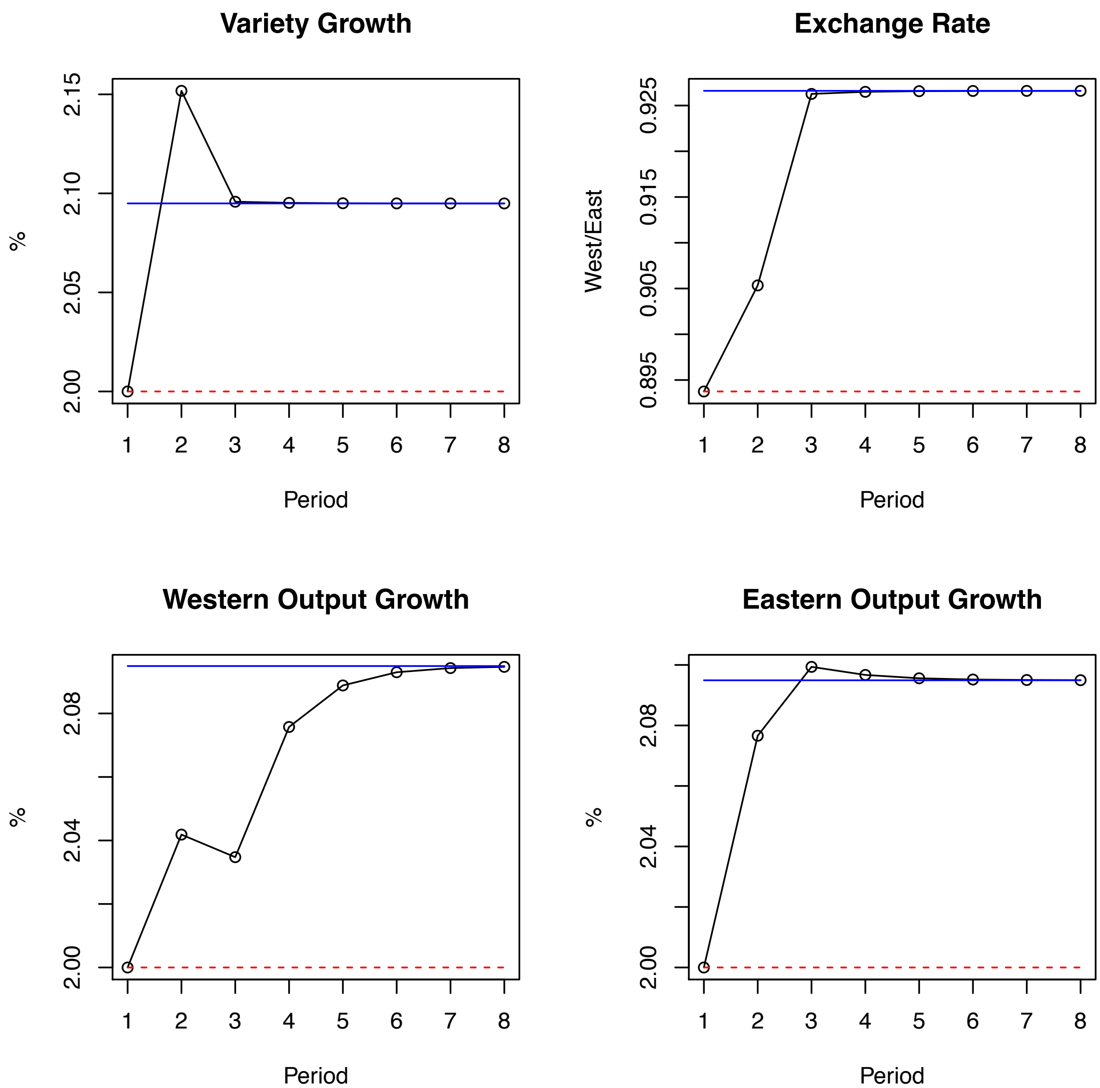

Figure 6: Trapped Factors Transition Dynamics, $\eta = 0.5$

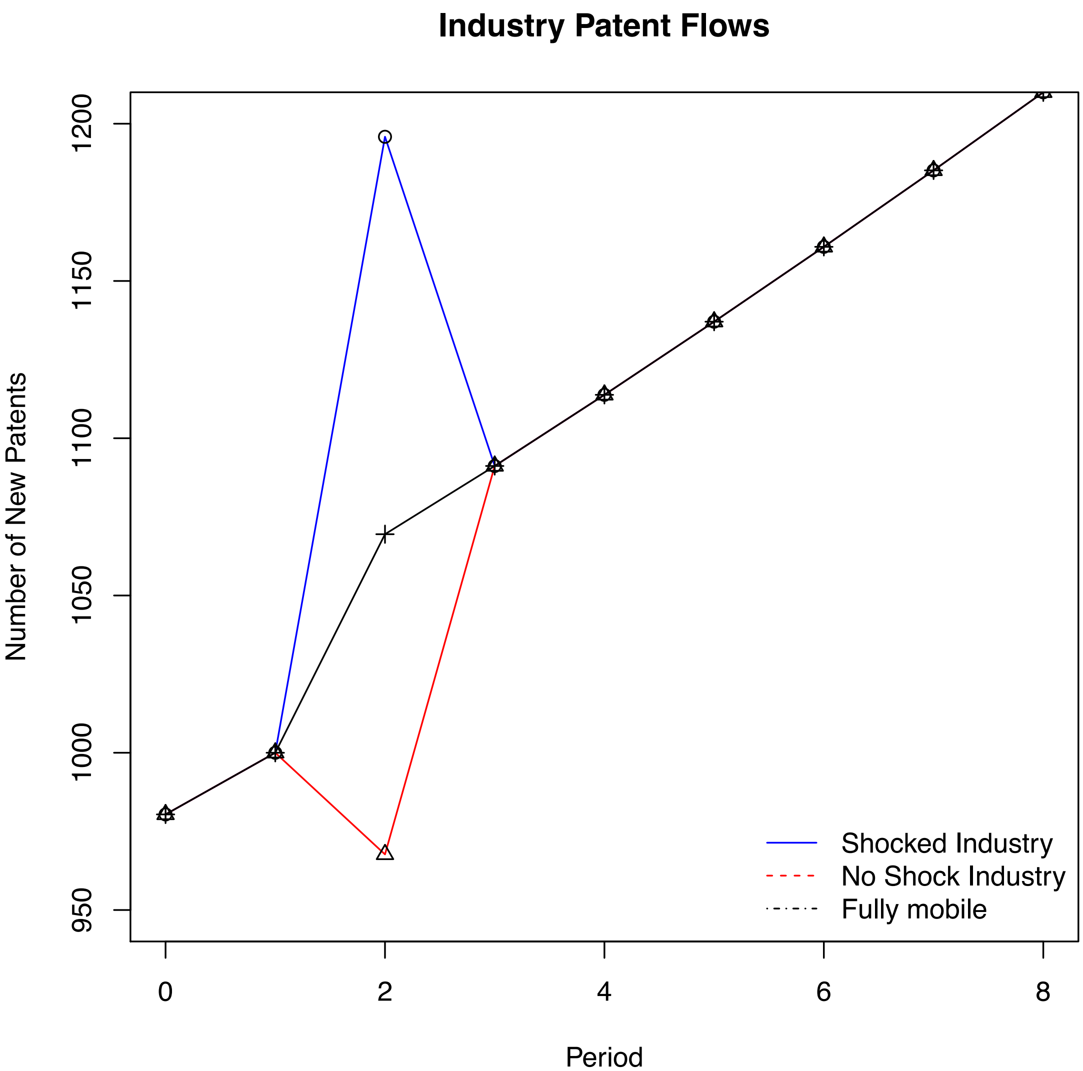

Figure 7: Cross-Sectional Impact of Trapped Factors, $\eta = 0$

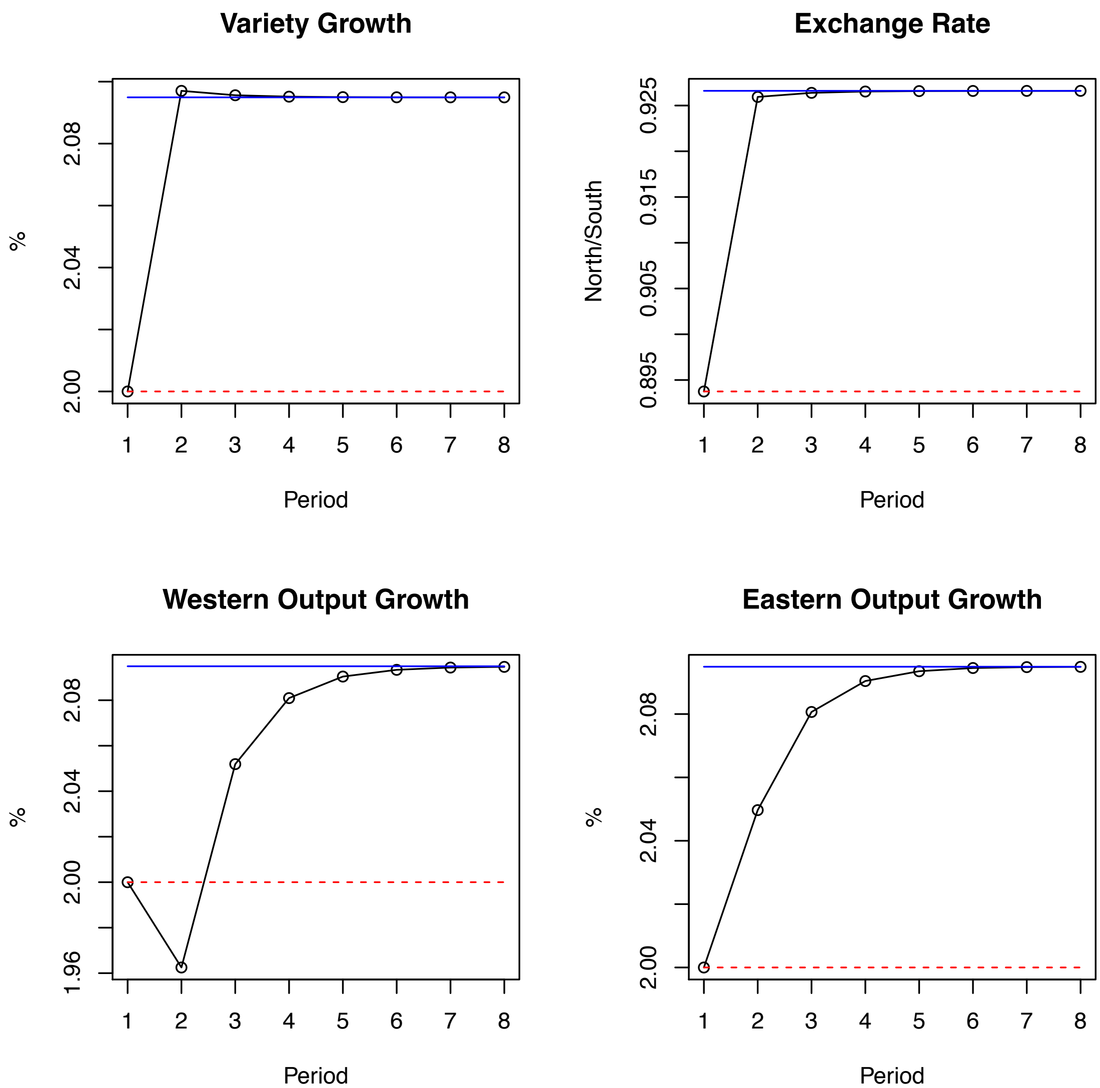

Figure 8: Trapped Factors Transition Dynamics, $\eta = 0$

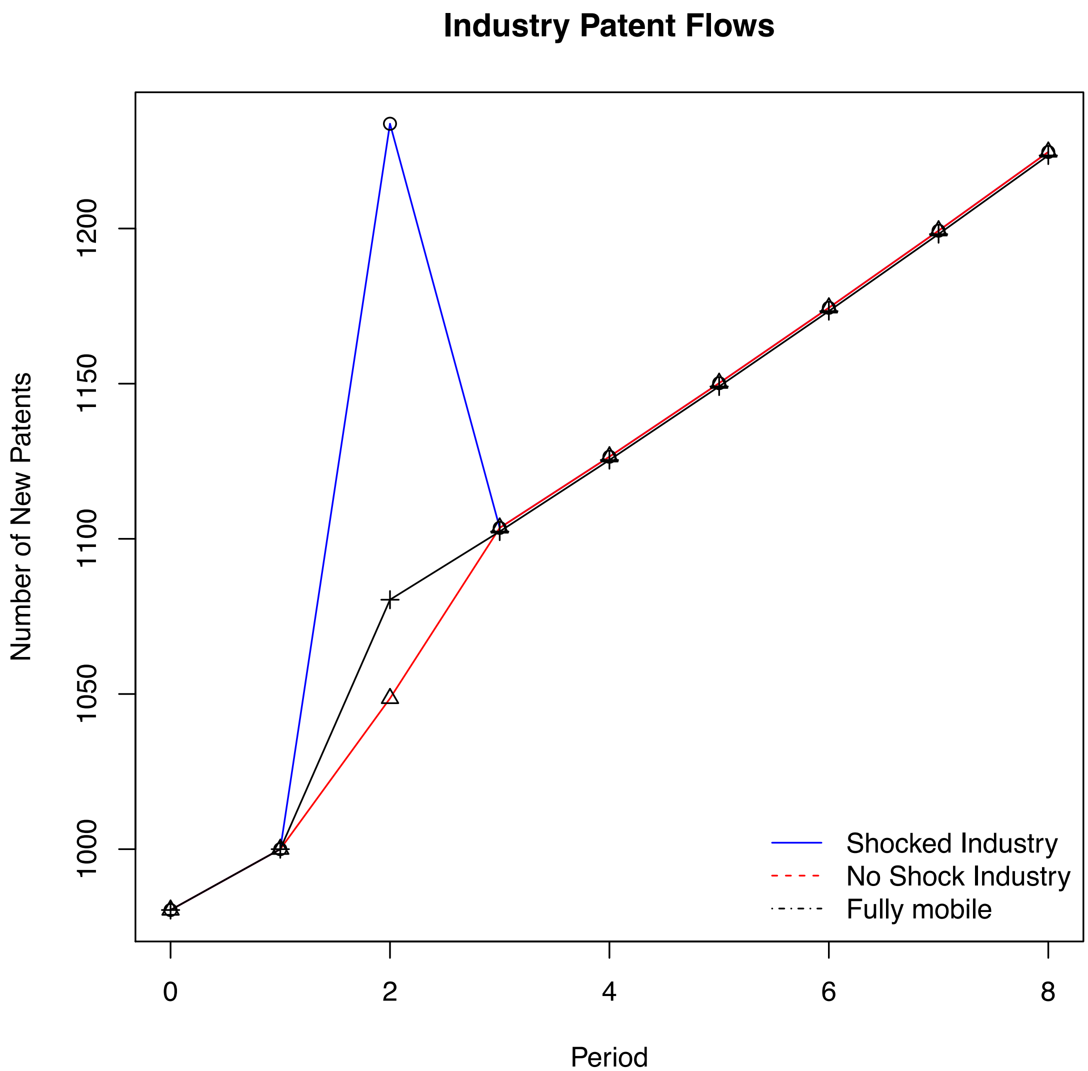

Figure 9: Cross-Sectional Impact of Trapped Factors, $\rho = 0.6$

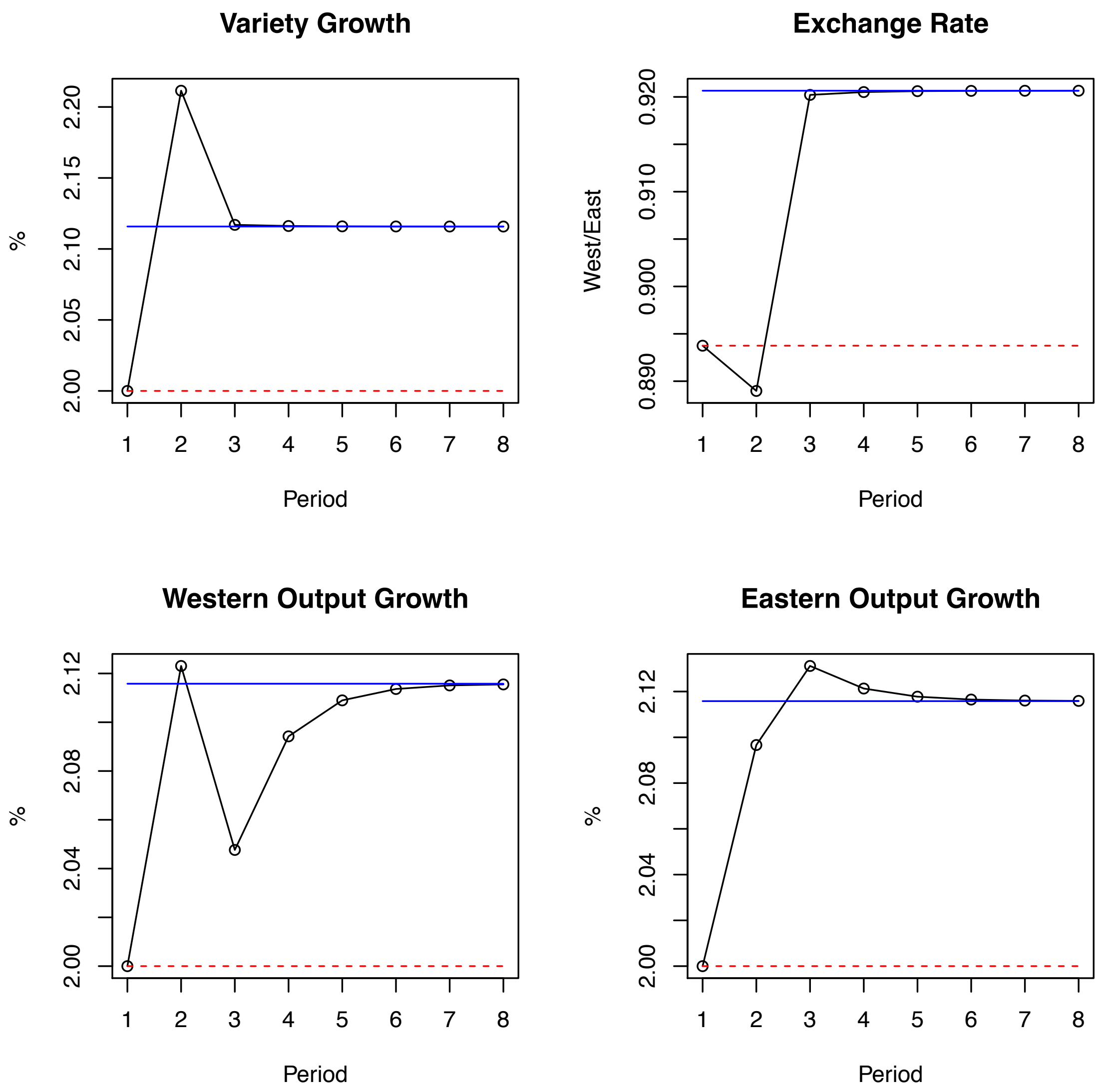

Figure 10: Trapped Factors Transition Dynamics, $\rho = 0.6$