

# The Risky Capital of Emerging Markets

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ABSTRACT

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Neoclassical theory predicts that the returns to capital should be equalized across countries. Yet, returns to capital in poor countries exceed the returns in rich ones. That capital does not flow from rich to poor countries to equalize the returns is termed the Lucas Paradox. In this paper we argue that higher average returns in poor countries compensate investors for higher levels of risk. We estimate a one-factor asset pricing model and find that risks associated with US investors' consumption growth can largely account for the cross-sectional dispersion in capital returns, albeit under high levels of risk aversion. Hence, the Lucas Paradox shares key features with well-known asset-pricing puzzles. We resolve the paradox within a two-country endowment framework where developed and emerging economies are exposed to different shocks to trend growth and investors exhibit recursive preferences. We find that differences in long-run risk can account for forty percent of the differences in returns to capital across developed and emerging markets.

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# 1. Introduction

Neoclassical theory predicts that the returns to capital should be equalized across countries, as this is the efficient allocation of capital and that which maximizes global output. Deviations from this outcome should be arbitrated away, and that they are not is generally interpreted as indicating the presence of frictions in international capital markets, and an opportunity to increase global output by reallocating the world capital stock. In his seminal paper, [Lucas \(1990\)](#) points out that the data reveal substantial and systematic dispersion in capital returns: poor countries exhibit significantly higher capital returns than do rich ones, begging the question of why capital does not flow from rich countries to poor until returns are equalized, as predicted by the theory. That observed disparities are large suggests that the potential gains from such a reallocation are substantial.

The prediction of return equalization, however, and the resulting implications for allocative efficiency in international capital markets, holds only in a deterministic setting. In the presence of uncertainty, asset pricing theory suggests that dispersion in rates of return should exist so long as there is dispersion in the riskiness of international capital holdings. In short, investors demand higher returns from riskier assets. Thus, incorporating risk into an analysis of international capital flows and capital returns delivers very different implications than in a deterministic setting: there is no reason to expect rate of return equalization, and indeed, if there are significant differences in the riskiness of international capital holdings, such an outcome would be surprising.

In this paper, we pursue this line of analysis by asking whether the risk-return tradeoff implied by consumption-based asset pricing theory can explain the observed dispersion in international capital returns, and in particular, the disparities between high return/poor countries and low return/rich ones. We find that the answer is yes. Consumption growth risk can account for much of the cross-sectional dispersion in capital returns, suggesting that the observed allocation of capital is almost exactly that predicted by theory in the presence of uncertainty, despite the existence of large return differentials. Differences in the riskiness of capital assets is thus a potential answer to the Lucas “paradox:” capital does not flow into high return/poor countries until returns are equalized precisely because these countries represent the riskiest investments. With this result in hand, we dig deeper and pinpoint the source of differential risk exposures of capital assets as stemming primarily from differences in long-run risks associated with investments in poor versus rich countries. Specifically, investors with recursive preferences demand higher returns from capital investments in low-income markets because cash flows streams from these investments not only exhibit larger shocks to trend growth, but also there is larger uncertainty surrounding the shocks to trend. We show that a suitably calibrated version of the long run risk model can quantitatively account for a significant portion of the difference in

capital returns between the US and a set of representative emerging markets.

We begin our analysis by laying out the asset pricing implications of a standard international stochastic growth model. Our focus is on a representative US investor able to invest in both domestic and international capital assets. The theory delivers asset pricing equations relating the expected returns to each asset to its consumption-based risk, captured by its covariance, or consumption beta, with US consumption growth. We then assess whether the theory can account for the cross-sectional dispersion in capital returns and in particular, for whether it can explain the high return/low-income versus low return/high-income pattern observed in the data. To do so, we follow common practice in empirical asset pricing and one that is spreading to macro-finance by constructing 8 portfolios of capital returns across countries grouped by income levels and using these portfolios as the primary unit of analysis. The portfolio approach serves to eliminate the idiosyncratic country-specific component of risk that investors should diversify away, enabling us to focus on the component of risk of interest: that related to income differences. In other words, we hope to isolate the part of risk associated with income and ask that our theory account for it and help uncover its source, rather than directly attack and account for the host of risk factors that determine the returns in any single country. This procedure creates a large spread in capital returns that is monotonically decreasing in income, confirming that capital returns are systematically related to income differences.

Our theory can explain the negative relationship between returns and income only to the extent that low-income portfolios exhibit greater risk for the US investor. We document that this is precisely the case. The consumption betas of the 8 portfolios are generally decreasing in income. Quantitatively, the dispersion in betas is significant and lines up closely with the dispersion in returns such that variation in risk explains much of the variation in capital returns across the 8 portfolios, a result we find striking. Restated another way, the observed allocation of the world's capital stock is in line with that predicted by theory once uncertainty is introduced, despite the existence of large rate of return differentials.

The estimated model, however, generates a high market price of risk and an associated level of risk aversion of almost 400. This result guides us to frame the Lucas paradox as simply "another asset-pricing puzzle." Motivated by the macro-finance literature that has been successful at accounting for a variety of asset pricing anomalies, such as the equity premium puzzle, we turn to an explanation that relies on long-run risks and a class of preferences that prices them. In particular, we postulate that US consumer preferences are recursive (as in [Kreps and Porteus \(1978\)](#), [Epstein and Zin \(1989\)](#), and [Weil \(1989\)](#)), which yield marginal rates of substitution that not only include the ratio of consumption in two consecutive periods, but also reflect changes in future welfare. Changes in future welfare are in turn driven by uncertainty surrounding the trend of the growth rate of cash flows that finance consumption. In fact, [Bansal and Yaron \(2004\)](#) demonstrate that long-run trend-growth risk largely accounts for the excessive returns

to risky capital in the US—namely, the equity premium.

We argue that the long run risks that stem from emerging market capital investments are larger than those associated with investments in the US. We are motivated by the observation made by [Aguiar and Gopinath \(2007\)](#) that shocks to trend growth are crucial in understanding economic fluctuations in developing markets. The key innovation is that we allow for different shocks to the trend growth rates of cash flows from developed and emerging markets, thereby capturing the different levels of risk inherent in these two asset classes. Keeping the parameters that govern the preferences of a typical US investor as well as the cash flows from investments in the US in line with the existing literature (and [Bansal and Yaron \(2004\)](#) in particular), we calibrate the parameters that govern the cash flows from emerging markets to match: (i) volatility of returns on invested capital in emerging markets relative to the US; and (ii) the correlation between returns on invested capital in emerging markets and the US.

The calibrated model yields differences in returns to capital between developed and emerging markets of more than four percent (even under quite reasonable levels of risk aversion). The corresponding statistic in post-1980 data amounts to ten percent, which suggests that differences in long-run risks can account for roughly forty percent of the observed differences in returns to capital across countries. In turn, implied capital-to-output ratios in emerging markets are roughly 0.7 times those in the US — a prediction that aligns almost perfectly with the data. Hence, we conclude that the observed cross-country capital allocation is consistent with the different levels of risk that stem from investments in emerging relative to developed countries.

## 2. Lucas Paradox: Just Another Asset-Pricing Puzzle

We begin this section by documenting that returns to capital in emerging markets exceed the returns in developed countries.

### 2.1. Measurement

We characterize the returns to capital in a standard international stochastic growth model. We describe the production economy only in brief as we are using it primarily as a measurement device.

The production technology in the final good sector in country  $j \in 1, \dots, J$  is

$$Y_{jt} = A_{jt} K_{jt}^{\alpha} L_{jt}^{1-\alpha}$$

such that countries employ identical production technologies, but may differ in their levels of productivities,  $A_j$ , and their factor levels. Let  $P_j$  denote the price of output in country  $j$ .

With this underlying production structure in mind, one can define the cash flow per unit of

capital (or the dividend) that an investor receives between periods  $t$  and  $t + 1$  from a claim to the capital stock of country  $j$  as

$$D_{jt+1} = \alpha \frac{P_{jt+1} Y_{jt+1}}{K_{jt+1}} \quad (1)$$

If capital depreciates at rate  $\delta$  every period, where  $\delta$  is common across countries, the gross return to the investor is given by

$$R_{jt+1} = \frac{P_{jt+1} + D_{jt+1} - \delta P_{jt+1}}{P_{jt}}. \quad (2)$$

Substituting (1) into (2) and rearranging yields

$$R_{jt+1} = \alpha \frac{Y_{jt+1}}{K_{jt+1}} \frac{P_{jt+1}}{P_{jt}} + (1 - \delta) \frac{P_{jt+1}}{P_{jt}}. \quad (3)$$

Assuming that consumption, investment, and output are perfectly tradable implies that  $P_{jt} = P_{j't}$  for all  $j \neq j'$ . Hence, we normalize each period's price to unity. The interpretation of expression (3) is as follows: The first term in (3) is the real return to current capital usage, which is the physical MPK, and the second term is the capital gain due to fluctuations in the value of the undepreciated portion of capital.

## 2.2. Risk and Returns

We consider a representative US investor who can invest in any country's capital as well as in a risk-free bond. Capital markets are frictionless. Let  $R_f$  denote the gross return to the risk-free asset. Then, the Euler equations (or no-arbitrage conditions) governing investment in the capital of country  $j$  and the risk free asset can be written as

$$1 = E_t [M_{t+1} R_{jt+1}] \quad (4)$$

$$1 = R_{ft+1} E_t [M_{t+1}] \quad (5)$$

where  $M_{t+1}$  is the US investor's stochastic discount factor (SDF). In the derivation above, we have used the fact that the risk-free return at date  $t + 1$  is known at  $t$ . It is straightforward to rearrange (4) and (5) to derive the following relationship between the returns to risky capital in country  $j$  and the safe asset

$$E_t R_{jt+1} - R_{ft+1} = -\frac{\text{cov}_t(M_{t+1}, R_{jt+1})}{E_t M_{t+1}} \quad (6)$$

which shows that the expected rate of return on the capital of country  $j$  relative to the risk-free rate at time  $t$ , i.e., the risk premium for country  $j$  capital assets, is determined by the covariance of the returns to capital in that country with the US investor's SDF. The intuition is standard: assets that covary negatively with marginal utility are risky because they pay off when marginal utility is low, exactly when investors would least like them to, and so demand a higher expected return. In the absence of uncertainty, (4) and (5) imply that capital returns are equalized across countries to the level of the risk-free rate, and thus that departures from such an allocation indicate the presence of distortions in international capital markets. In our stochastic environment, there is no reason this outcome should hold, and in fact, it would be surprising if it did, given any significant dispersion in risk exposure across countries.<sup>1</sup>

**A Simple Measure of Risk Under CRRA Preferences** The risk premium expression derived above holds for any specification of consumer preferences. In order to relate the stochastic discount factor to data, however, a stand on preferences is needed.

We begin by assuming that a representative US investor seeks to maximize the expected discounted flow of utility given by

$$E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t U(c_t)$$

where  $c$  denotes per-capita consumption of a single homogeneous good,  $\tilde{\beta}$  is the investor's discount factor, and  $E$  is the expectation operator conditional on information available at date 0 (the present time). Preferences take the constant relative risk aversion (CRRA) form

$$U(c) = \frac{c^{1-\gamma} - 1}{1-\gamma},$$

where  $\gamma \geq 0$  is the risk aversion coefficient.

Under this preference specification, the SDF becomes  $M_{t+1} = \tilde{\beta} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$ . Substituting the SDF into equation (6) shows that dispersion in the returns to capital can be explained by the CRRA model only to the extent that there is variation in exposure to US consumption risk. Moreover, to reconcile the high return/low-income versus low return/high-income pattern observed in the data, this variation must be systematically related to income: emerging markets must exhibit higher risk than developed ones. We next document that this is precisely the case: dispersion in risk, captured by the covariance of returns with the SDF of the US investor, is able to account for a large portion of the dispersion in international capital returns as well as address the puzzle of why capital doesn't flow from rich to poor.

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<sup>1</sup>Moreover, the fact that measured capital returns substantially exceed the risk-free rate suggests that such investments are risky for investors.

### 2.3. Risk and Returns to International Capital

In this section, we use the simple CRRA framework derived above to ask whether US consumption growth risk can explain variation in the returns to capital. The net return to capital in market  $j$  is given by

$$r_{jt+1} = \alpha \frac{Y_{jt+1}}{K_{jt+1}} - \delta. \quad (7)$$

To measure net capital returns we use data on  $Y$  and  $K$  from version 7.1 of the Penn World Tables (Heston, Summers, and Aten (2012)). We set  $\alpha = 0.3, \delta = 0.06$  which are standard values, and use equation (7) to construct returns by country and year. Our approach follows closely the literature and we leave the details to Appendix A.<sup>2</sup> Our final sample contains 110 countries over the period 1960 to 2009.

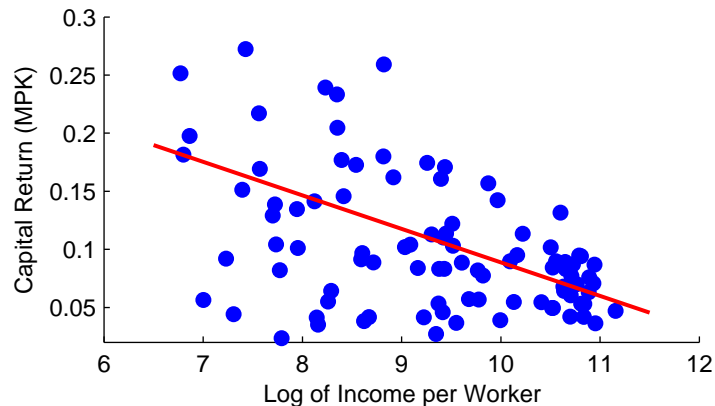


Figure 1: The Cross-Section of Capital Returns

Figure 1 plots the return to capital for each country in our sample against the log of income per worker. Both statistics are averaged over the 50-year period. Poor countries tend to exhibit significantly higher rates of return to capital than do rich ones, as highlighted by the negative slope of the line of best fit. The slope of the line amounts to  $-0.04$  and is statistically significant at the 5-percent level.

**Portfolio construction.** The puzzle we are after is why capital doesn't flow from low return/rich countries to high return/poor ones. In particular, we will shortly reformulate this question as follows: Can differences in the riskiness of capital explain the dispersion in capital returns across countries with varying income levels?

To focus on the link between capital returns and income on the one hand, and risk on the other, we build 8 portfolios of capital returns across countries grouped by levels of income. We use

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<sup>2</sup>See for example, Caselli (2005), Caselli and Feyrer (2007), and Gourinchas and Jeanne (2011).

these portfolios, rather than individual countries, as the primary unit of empirical analysis in this section. The portfolio approach is standard in empirical asset pricing, which has generally moved from addressing variation in individual asset returns to returns on asset portfolios, sorted by factors that are known to predict returns. This procedure proves useful in eliminating asset-specific diversifiable risk, and so in honing in on the sources of return variation of interest. In our application, it serves to eliminate idiosyncratic factors driving country-specific returns that are unrelated to their levels of economic development.<sup>3</sup> Moreover, we are able to expand the number of countries as data becomes increasingly available, enabling us to include the largest possible set of countries in our analysis. Lastly, we feel that there is an intuitive appeal to analyzing portfolios: by doing so, we are asking whether there are arbitrage opportunities for an investor to go short in a portfolio of rich country capital assets and long in a portfolio of poor country ones, which is at the heart of the question we are after. This approach has recently been adopted by the macro-finance literature, with a key example being [Lustig and Verdelhan \(2007\)](#), and we follow their lead closely. We follow [Lustig and Verdelhan \(2007\)](#) in our choice of 8 portfolios, as we find similarly to them that increasing the number beyond this subjects the performance of some portfolios to the influence of outliers. Our empirical results are quite robust to the number of portfolios chosen.

In each year then, we allocate countries into one of 8 portfolios sorted by income. For example, portfolio 1 contains the poorest set of countries in that year and portfolio 8 the richest, so that higher numbered portfolios are higher income, a terminology which will remain consistent throughout the paper. Portfolios are rebalanced every year. We compute the returns to capital in each portfolio as the average over the component countries. Recall our goal here is to isolate the variation in capital returns that is related to differences in income and eliminate the country-specific idiosyncratic component of returns.

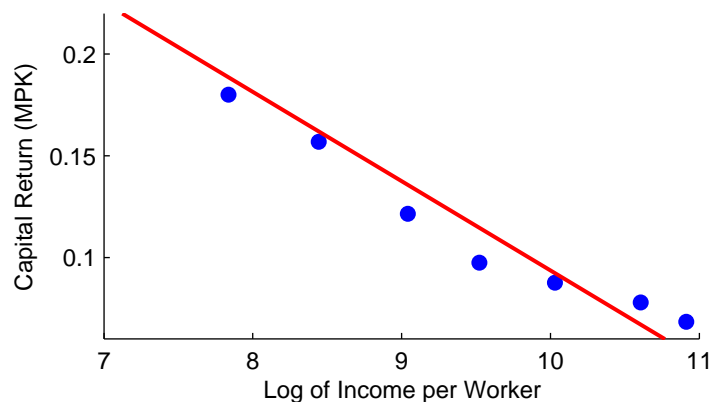


Figure 2: The Cross-Section of Capital Returns

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<sup>3</sup>The portfolio approach also aids in eliminating measurement error in country-level variables (for example, in the construction of capital stocks) and in errors introduced from relying on empirical estimates of covariances, rather than actual.



The results are shown in Figure 2, which displays returns to capital by income for each of the 8 portfolios, along with the line of best fit. There is a large spread in the cross-sectional returns to capital, ranging from 7.6% in portfolio 8, the richest, to 18% in portfolio 1, the poorest. The relationship between income and returns is very close to linear in the log of income and monotonically decreasing, confirming that our procedure produces dispersion in returns that are systematically related to income. Table 1 reports the mean return for each portfolio, along with the simple measure of risk under CRRA preferences, which we revisit below.

**Linear model.** To assess the ability of the theory to explain dispersion in capital returns, we follow the literature and derive a linear relationship between expected excess returns and the covariance of returns with log consumption growth, that is, we write a linear factor model for excess returns. To do so, we linearize the SDF around its unconditional mean as  $M_{t+1} = (E[M_{t+1}]) (1 + m_{t+1} - E[m_{t+1}])$  where  $m_t = \log M_t$ , which, along with the definition of  $m_t$ , can be used with the unconditional expectation of (6) to find

$$E[R_{jt}^e] = \gamma \text{cov}(\Delta c_t, R_{jt}) \quad (8)$$

where  $R_{jt}^e = R_{jt} - R_{ft}$  denotes excess returns at time  $t$  and  $\Delta c_t$  the log of US consumption growth. This gives a beta pricing model

$$E[R_{jt}^e] = \beta_j \lambda \quad (9)$$

where the consumption beta of portfolio  $j$  captures its riskiness and is defined as

$$\beta_j = \frac{\text{cov}(\Delta c_t, R_{jt})}{\text{var}(\Delta c_t)} \quad (10)$$

and  $\lambda = \gamma \text{var}(\Delta c_t)$  is the price of consumption-growth risk. Equation (9) is the main object of interest: for a given level of the price of risk  $\lambda$ , can variation in consumption betas explain variation in capital returns? In light of (9), our theory rests upon the idea that consumption betas vary across portfolios in a systematic way, and that the relation between returns and betas are close to linear. We proceed by first constructing the consumption betas implied by equation (10) using the time-series data on US consumption growth and portfolio returns and then running the regression implied by (9), where the  $R^2$ , representing the amount of cross-sectional variation in returns explained by consumption growth risk, will measure the success or failure of the theory.<sup>4</sup>

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<sup>4</sup>Our approach is essentially in the spirit of Fama and MacBeth (1973) and has been recently employed by Lustig and Verdellhan (2007) in a related application, among a large multitude of others.

## 2.4. Cross-Sectional Analysis

Table 1: Portfolio Risk and Returns under CRRA Preferences

Portfolio	1	2	3	4	5	6	7	8
Excess Returns	0.180	0.158	0.150	0.117	0.097	0.091	0.082	0.076
Standard Deviation	0.140	0.116	0.112	0.100	0.071	0.087	0.074	0.075
Consumption Beta	2.540	2.411	2.005	1.381	0.663	0.919	1.089	0.758
Standard Error	(1.405)	(1.150)	(1.127)	(1.018)	(0.729)	(0.888)	(0.747)	(0.771)

Our first step is to use (10) to compute portfolio consumption betas. We report the results in Table 1. There is significant variation in betas across portfolios, ranging from a high of 2.54 to a low of 0.758, a factor of more than 3. Moreover, although not perfect, there is a clear negative relationship between these betas and income, implying that capital investments in poor countries are riskier than in rich countries. The patterns of international returns then are at least qualitatively in line with the theory: high risk capital investments in poor countries demand higher returns than do low risk investments in rich ones. In addition, returns in poor countries are on average more volatile as can be seen from the higher standard deviations surrounding the lower numbered portfolios.

Table 2: Second-Pass Estimate of Factor Price under CRRA Preferences

	Consumption Factor	$\gamma$	$R^2$	p-value	MAE
$\lambda$	0.075	<b>399.815</b>	0.645		0.017
SE	(0.007)			0.000	

Our next step is to perform the regression suggested by (9), from which the  $R^2$  tells us how much of the cross-sectional variation in excess returns across portfolios is explained by variation in consumption betas. The  $R^2$  amounts to 0.645, which suggests that more than half the variation in portfolio returns is explained by consumption growth risk. Thus a possible answer to [Lucas \(1990\)](#) is: Capital doesn't flow from rich to poor countries to equalize returns because capital investments in poor countries are riskier and demand higher returns, in line with the predictions of asset pricing theory.

**The Elephant in the Room.** There are two caveats with the empirical analysis above. The first is apparent in Table 2: the market price of risk predicted by the model is high and implies a level of risk aversion  $\gamma$  of about 400. This result is simply the equity premium puzzle in another guise. Generating sufficient volatility in the stochastic discount factor to match the levels of risk premia in capital markets requires an unreasonable level of risk aversion. However, our focus

here is on whether there is enough variation in consumption and market betas to explain the dispersion in returns, conditional on a level of risk aversion large enough to match the levels of excess returns, a question we answer in the affirmative. We thus follow studies such as [Lustig and Verdelhan \(2007\)](#) and [Yogo \(2006\)](#) in interpreting our results as showing that given the degree of risk aversion needed to match the level of excess returns to capital, a risk-based explanation can reconcile the spread in returns across various capital assets.<sup>5</sup> This is the focus of the next step of our analysis, where we place a bit more structure on the problem and ask whether a deeper theory of the sources of risk can generate the observed differences in capital returns under more reasonable levels of risk aversion.

Second, we do not include a constant term when computing the model’s predicted excess returns across portfolios because the theoretical pricing equation implies a zero intercept. However, the standard practice in the literature is to include a constant and interpret it as a pricing error that is common across all assets.<sup>6</sup>

Table 3: Second-Pass Estimates of Factor Price with Constant under CRRA Preferences

	Constant	Consumption Factor	$\gamma$	$R^2$	p-value	MAE
$\lambda$	0.045	0.050	<b>266</b>	0.922		0.008
SE	(0.010)	(0.007)			0.000	

Table 3 reports the results from the second-stage regression that includes a constant term. The constant term is positive and statistically significant. The  $R^2$  increases to a remarkable 92 percent. The implied risk aversion coefficient falls to 266, but is nonetheless in excess of any reasonable estimate of the parameter in macro studies. Hence, adding a constant aids in putting the model’s predictions on the average level of excess returns observed in the data, but, conditional on that level, has little impact on the predicted dispersion across capital assets, our primary object of interest.

**Relation to [Caselli and Feyrer \(2007\)](#).** In Appendix B, we characterize the returns to capital in a two-sector international stochastic growth model. We are motivated by the existing literature that has suggested an important role of relative prices in determining capital returns, for example [Caselli and Feyrer \(2007\)](#) and [Hsieh and Klenow \(2007\)](#), as well as of capital gains

<sup>5</sup>For example, [Lustig and Verdelhan \(2007\)](#) find a risk aversion of 114 and [Yogo \(2006\)](#) report one close to 200. The level we find in an international setting is not surprising, given the results in [Campbell \(2003\)](#), who shows that explaining the equity premium puzzle internationally requires levels of risk aversion as high as 1713 (and as low as essentially zero).

<sup>6</sup>For example, [Burnside \(2011\)](#) notes that a positive constant, as we find below, can be attributable to measurement errors in estimated betas or to a liquidity premium in T-bills. Inclusion of a constant is in line with the majority of asset pricing studies of dispersion in returns, a non exhaustive list of recent work includes [Lustig and Verdelhan \(2007\)](#), [Yogo \(2006\)](#), and [Parker and Julliard \(2005\)](#).

in determining their dynamic behavior, for example [Gomme, Ravikumar, and Rupert \(2011\)](#). We compute capital returns in the data with this setting in mind and we show our findings in Appendix B. All of our results are robust to this specification.

Why do we find substantial differences in capital returns across countries where [Caselli and Feyrer \(2007\)](#) do not? A key difference in our approach is that we examine the behavior of returns across 50 years while they focus on a single year 1996. In this light, that returns are approximately equalized in a given year is not inconsistent with our analysis. In our framework, returns in a particular year are simply a single realization of a stochastic process which may at times lead to ex-post returns that are quite similar and at other times quite different. As an example, we find similarly that in 1996, the negative relationship between returns and income is almost completely absent.<sup>7</sup> However, by the mid-2000s, this is no longer the case and we find a significant negative relationship. In 1995, the relationship is negative and significant whereas in 1990 it is not. The key to our analysis is not whether returns happen to be equalized or not in a particular year, but whether the realizations of the stochastic process governing returns lead to covariances with US consumption growth that differ systematically with income, such that poor countries pose greater risk than rich, a question we answer in the affirmative.

### 3. Resolving the Lucas Paradox

In the previous section, we showed that the Lucas Paradox shares common features with standard asset pricing puzzles in the finance literature. We turn to this literature in an attempt to reconcile the puzzle. The first lesson from the asset pricing literature is that observed excess returns in the data can be reconciled only if the SDF is highly volatile. This result is known as the Hansen-Jagannathan bound. With CRRA preferences, which tie the volatility of the SDF to consumption growth, the Hansen-Jagannathan lower bound can only be satisfied if the risk aversion coefficient is unreasonably high. The observation suggests that the measure of risk that stems from the simple CRRA specification is not satisfactory. This brings us to a second insight of the asset pricing literature: recursive preferences in conjunction with long-run risks offer a satisfactory measure to risk. The latter specification offers a multi-factor asset pricing model where asset returns are sensitive not only to period-by-period fluctuations in consumption, but also to consumption trend growth and uncertainty about the trend growth.

[Bansal and Yaron \(2004\)](#) rely on such multi-factor model and show that long-run trend growth risk is key to account for the excessive returns to capital in the US over a long period of time—namely, the equity premium puzzle. Can the pricing of long-run risks reconcile the Lucas puzzle as well? In order to answer this question in the affirmative, it is necessary that long-run

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<sup>7</sup>Regressions of the former on the latter across portfolios or countries yield small negative coefficients that are not significantly different from zero at standard confidence levels.

risks are even larger and more important in poor countries than in rich countries. This line of argument is consistent with [Aguiar and Gopinath \(2007\)](#), who demonstrate that shocks to trend growth are larger in poorer countries—namely, the cycle is the trend in emerging markets.

Consequently, we attempt to reconcile the Lucas Paradox by incorporating long-run risks into a multi-factor asset pricing model. We ask the following two questions: (i) Can differences in (long-run) risk account for the observed differences in returns to capital across rich and poor countries when investors are reasonably risk averse? (ii) Can differences in risk reconcile observed capital allocations across countries?

### 3.1. A Model with Long-Run Risk

We assume that the US investor's preferences are recursive as in [Kreps and Porteus \(1978\)](#), [Epstein and Zin \(1989\)](#), and [Weil \(1989\)](#), and are given by

$$V_t = \left[ \left(1 - \tilde{\beta}\right) C_t^{\frac{\psi-1}{\psi}} + \tilde{\beta} \nu_t^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}, \quad \psi \geq 0$$

where

$$\nu_t(V_{t+1}) = [E_t(V_{t+1}^{1-\gamma})]^{\frac{1}{1-\gamma}}, \quad \gamma \geq 0$$

is the certainty equivalent function,  $\psi$  is the intertemporal elasticity of substitution,  $\gamma$  is the coefficient of risk aversion, and  $\tilde{\beta} < 1$  is the discount rate. Defining  $\theta = (1 - \gamma)\psi/(\psi - 1)$ , standard arguments give the stochastic discount factor as

$$M_{t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left( \frac{V_{t+1}}{\nu_t[V_{t+1}]} \right)^{\theta-1}$$

The corresponding logged counterpart is particularly useful for quantitative analysis,

$$\log m_{t+1} = \log \delta - \underbrace{\frac{\theta}{\psi} \log(C_{t+1}/C_t)}_{\text{short-run risk}} + (\theta - 1) \underbrace{(\log V_{t+1} - \log \nu_t[V_{t+1}])}_{\text{long-run risk}} \quad (11)$$

Expression (11) shows that, according to this preference specification, investors do not only want to smooth consumption, but also their entire future welfare.

The problem with expression (11) above is that the long-run risk component is not readily available in the data. While it can potentially be quantified using long data series for developing countries such as the US (see for example [Nakamura, Sergeyev, and Steinsson \(2012\)](#)), data limitations for developing countries make the exercise nearly impossible. Consequently, below we follow [Bansal and Yaron \(2004\)](#) in that we specify processes for the cash flows from emerging and developed markets and we use data on returns to productive capital (the stock market)

in order to infer long run risks.

We model the US investor's consumption process following [Bansal and Yaron \(2004\)](#):

$$\begin{aligned}\Delta c_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1} \\ x_{t+1} &= \rho x_t + \varphi_x \sigma_t \varepsilon_{t+1} \\ \sigma_{t+1}^2 &= \sigma_0^2 + \nu(\sigma_t^2 - \sigma_0^2) + \sigma_\omega \omega_{t+1}\end{aligned}$$

where the shocks  $\eta_{t+1}, \varepsilon_{t+1}, \omega_{t+1} \sim NIID(0, 1)$ . For simplicity, all shocks are uncorrelated. In the expression above,  $\mu$  represents the unconditional mean growth rate of consumption.  $x_t$  is a time-varying trend, where  $\rho$  is persistence in the trend component.  $\sigma_t$  and  $\varphi_x$  scale the volatility of the consumption process and the trend component, respectively.

We introduce shocks to trend to the growth rate of the dividend as well. For any asset  $j$  we specify the following process

$$\Delta d_{j,t+1} = \mu_{d,j} + \phi_{d,j} x_t + \varphi_{d,j} \sigma_t u_{j,t+1}$$

where  $u_{j,t+1} \sim NIID(0, 1)$ . In the above expression,  $\mu_{d,j} = \mu$  for all  $j$  is the unconditional mean growth rate of the dividend, averaged across assets. We keep this parameter common across assets in order to ensure that the dividend processes of different (countries') assets do not diverge.  $\phi_{d,j}$  measures the asset's exposure to long-run risk in US consumption growth, and  $\varphi_{d,j}$  scales the volatility of the dividend process.

### 3.2. Calibration

We focus on two economies in the post 1980-period: the US and a set of emerging markets that comprise the MSCI index.

We choose parameters that govern the US investor's preferences, consumption growth and cash flow growth from the existing literature and [Bansal and Yaron \(2004\)](#) in particular.

What remains is to calibrate the parameters that govern the cash flow processes for the emerging markets. We consider a set of emerging markets that constitute the MSCI index between 1986 and 2013. We calibrate  $\phi_{d,em}$  and  $\varphi_{d,em}$  to match: (i) the volatility of returns on invested capital in the emerging markets, 0.069, relative to the US, 0.042 ; and (ii) the correlation of returns on invested capital in the emerging markets and the US, which amounts to 0.67.

Table 4 summarizes the parameter values.

Table 4: Baseline Calibration

$\delta$	Time-preference parameter	.999
$\psi$	Intertemporal rate of substitution	1.5
$\gamma$	Long-run risk aversion	10
$\mu$	Ergodic mean of consumption and dividend growth	0.0015
$\rho$	Persistence parameter for $x$	0.979
$\varphi_x$	Volatility scale for $x$	0.044
$\sigma_0$	Ergodic mean of $\sigma$	0.0078
$\nu$	Persistence parameter for stochastic vol.	0.987
$\sigma_w$	Volatility scale for volatility	$0.25 \cdot 10^{-5}$
$\phi_{d,us}$	Sensitivity to growth shocks $x$	3
$\varphi_{d,us}$	Volatility scale for US dividend growth	2.5
$\phi_{d,em}$	Sensitivity to US growth shocks $x$	6.025
$\varphi_{d,em}$	Volatility scale for EM dividend growth	5.1

### 3.3. Discussion of Results

The calibrated model yields a difference in the returns between the portfolio of emerging and developed countries' capital of 4.03%. The corresponding difference in the MSCI and US stock market data is 10%. Hence, differences in risk can account for roughly 40% of the differences in the rates of return on capital between rich and poor countries.

### 3.4. Implications About World Capital Allocation

Using the predicted differences in returns from the calibrated model and the simple measurement structure we introduced in Section 2.1, we can compute the implied capital allocation across developed and emerging markets.

Modulo depreciation, the return on capital satisfies

$$R_{t+1} \approx \frac{P_{t+1} + \alpha \frac{Y_{t+1}}{K_{t+1}}}{P_t}.$$

Taking unconditional expectations yields

$$E(R) \approx \alpha \frac{Y}{K}.$$

Hence, the predicted differences in returns to capital imply that the predicted optimal capital

allocations must satisfy

$$\alpha \left(\frac{K}{Y}\right)_{em}^{-1} - \alpha \left(\frac{K}{Y}\right)_{us}^{-1} = E(R_{em}) - E(R_{us})$$

Taking a standard value for the labor share,  $\alpha = 1/3$ , and the mean US capital-to-output ratio post-1980,  $K/Y = 3$  gives

$$\left(\frac{K}{Y}\right)_{em} = \frac{1}{\left(\frac{K}{Y}\right)_{us}^{-1} + \frac{1}{\alpha} [E(R_{em}) - E(R_{us})]} = 2.20$$

Hence, the predicted emerging-market capital-to-output ratio amounts to roughly 70 percent of that in the U.S. This prediction is consistent with observations reported by [Hsieh and Klenow \(2010\)](#). Thus, differences in risk alone can account for a significant portion of the observed cross-country allocation of capital.

### 3.5. Future Work

One caveat of the benchmark calibration is that the implied volatility of the cash flows from capital investment in emerging markets is far in excess of the volatility observed in the PWT data. In order to address this problem, future robustness exercises will incorporate more flexible specifications of the cash flow processes for the two markets.

## 4. Conclusion

Neoclassical theory predicts that the returns to capital should be equalized across countries. Yet, returns to capital in poor countries exceed the returns in rich ones. That capital does not flow from rich to poor countries to equalize the returns is termed the Lucas Paradox. In this paper we argue that higher average returns in poor countries compensate investors for higher levels of risk. We estimate a single factor asset pricing model and find that consumption-based risk can largely account for the cross-sectional dispersion in capital returns, albeit under high levels of risk aversion. Hence, the Lucas Paradox shares key features with well-known asset-pricing puzzles. We resolve the paradox within a two-country endowment framework where rich and poor economies are exposed to different shocks to trend growth and investors exhibit recursive preferences. We find that differences in long-run risk can account for forty percent of the differences in returns to capital across rich and poor countries.

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## A. Data

**Risk-free rate.** 1 month T-bill from Ibbotsons. Downloaded from WRDS Fama-French factors.

**Consumption of Nondurables and Services.** From Personal Consumption Expenditures downloaded from BEA. Nominal expenditures from Table 2.3.5 on nondurables and services are deflated respectively by their price indexes in Table 2.3.4 and then added together to obtain real total expenditures on nondurables and services.

**CPI.** All urban consumers, seasonally adjusted, downloaded from FRED.

## B. Measurement — Two-Sector Model

The production technologies in the consumption and investment good sectors in country  $j \in 1, \dots, J$  are

$$\begin{aligned} C_{jt} &= A_{Cjt} K_{Cjt}^\alpha L_{Cjt}^{1-\alpha} \\ I_{jt} &= A_{Ijt} K_{Ijt}^\alpha L_{Ijt}^{1-\alpha} \end{aligned}$$

such that countries employ identical production technologies, as do sectors, but may differ in their levels of sector-specific productivities,  $A_{Cj}$  and  $A_{Ij}$ , and their factor levels. Aggregate factor endowments satisfy  $L_j = L_{Cj} + L_{Ij}$  and  $K_j = K_{Cj} + K_{Ij}$ , and the value of total output  $Y_j$  is given by

$$P_{Yjt} Y_{jt} = P_{Cjt} C_{jt} + P_{Ijt} I_{jt},$$

where  $P_{Yj}$ ,  $P_{Cj}$ , and  $P_{Ij}$  denote the prices of country  $j$  output, consumption, and investment, respectively.

With this underlying production structure in mind, one can define the dividend that an investor earns between periods  $t$  and  $t + 1$  from a claim to the capital stock of country  $j$  (or the cash flow per unit of capital) as

$$D_{jt+1} = \alpha \frac{P_{Yjt+1} Y_{jt+1}}{K_{jt+1}} \quad (12)$$

If capital depreciates at rate  $\delta$  every period, the gross return to the investor is given by

$$R_{jt+1} = \frac{P_{Ijt+1} + D_{jt+1} - \delta P_{Ijt+1}}{P_{Ijt}}, \quad (13)$$

where we have exploited the fact that capital is freely transferred between sectors and so the value of marginal products is equated across sectors and so to that of total production.

Substituting (12) into (13) and rearranging yields

$$R_{jt+1} = \alpha \frac{Y_{jt+1}}{K_{jt+1}} \frac{P_{Yjt+1}}{P_{Ijt+1}} + (1 - \delta) \frac{P_{Ijt+1}}{P_{Ijt}}. \quad (14)$$

The first term in (14) is the real return to current capital usage, which is the physical MPK corrected for the relative price of output and investment goods, and the second term is the capital gain due to fluctuations in the value of the undepreciated portion of capital. Note that this collapses to the MPK plus undepreciated capital in a one-sector model  $\alpha Y_{jt+1}/K_{jt+1} + 1 - \delta$  as the relative price is always one and there are no capital gains.

To measure capital returns we use data on  $Y$ ,  $K$ ,  $P_Y$ , and  $P_I$  from version 7.1 of the Penn World Tables (Heston, Summers, and Aten (2012)). We set  $\alpha = 0.3$ ,  $\delta = 0.06$  which are standard values, and use the net returns that correspond to the gross return in equation (3) to construct returns by country and year. Clearly, returns to capital are higher in poorer countries. The slope of the line of best fit in Figure 3 is -0.03 and is statistically significant at the 5-percent level. Figure 4, which groups countries into the same 8 portfolios discussed in the text, captures the striking negative link more clearly.

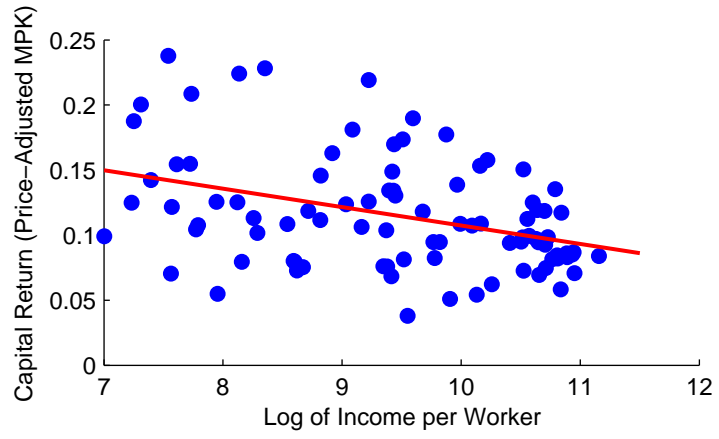


Figure 3: The Cross-Section of Capital Returns

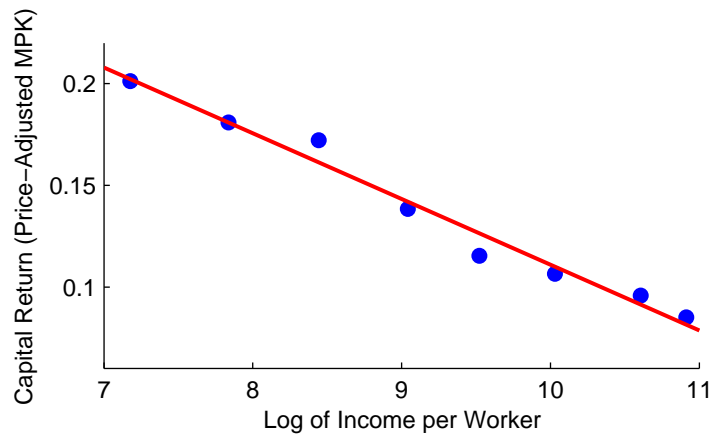


Figure 4: The Cross-Section of Capital Returns